

# Package ‘fitODBOD’

November 20, 2016

**Type** Package

**Title** Modeling over dispersed Binomial Outcome Data using Binomial Mixture Distributions and Alternate Binomial Distributions.

**Version** 1.0.0

**Description** Contains pdf,cdf,moment about zero values for mixing distributions and pmf, cmf, negative log likelihood value, parameter estimation and modeling data.

**License** GPL-2

**LazyData** TRUE

**RoxxygenNote** 5.0.1

**Imports** bbmle, hypergeo, knitr, rmarkdown

**Author** Amalan Mahendran [aut, cre],  
Prof.Pushpa Wijekoon [aut]

**Maintainer** Amalan Mahendran <amalan0595@gmail.com>

## R topics documented:

Alcohol_data . . . . .	3
BODextract . . . . .	3
Chromosome_data . . . . .	4
Course_data . . . . .	5
dAddBin . . . . .	5
dBETA . . . . .	7
dBetaBin . . . . .	9
dCorrBin . . . . .	11
dGBeta1 . . . . .	14
dGHGBB . . . . .	16
dGHGBeta . . . . .	18
dKUM . . . . .	20
dKumBin . . . . .	22
dMcGBB . . . . .	24
dMultiBin . . . . .	26
dTRI . . . . .	27
dTriBin . . . . .	30
dUNI . . . . .	32
dUniBin . . . . .	33
EstMGFBetaBin . . . . .	35
EstMLEAddBin . . . . .	36

EstMLEBetaBin . . . . .	38
EstMLECorrBin . . . . .	39
EstMLEGHGBB . . . . .	41
EstMLEKumBin . . . . .	42
EstMLEMcGBB . . . . .	44
EstMLEMultiBin . . . . .	45
EstMLETriBin . . . . .	47
Exam_data . . . . .	48
fitAddBin . . . . .	49
fitBetaBin . . . . .	50
fitBin . . . . .	52
fitCorrBin . . . . .	53
fitGHGBB . . . . .	55
fitKumBin . . . . .	56
fitMcGBB . . . . .	58
fitMultiBin . . . . .	60
fitTriBin . . . . .	61
mazBETA . . . . .	63
mazGBeta1 . . . . .	65
mazGHGBeta . . . . .	67
mazKUM . . . . .	69
mazTRI . . . . .	71
mazUNI . . . . .	73
NegLLAddBin . . . . .	75
NegLLBetaBin . . . . .	76
NegLLCorrBin . . . . .	77
NegLLGHGBB . . . . .	78
NegLLKumBin . . . . .	80
NegLLMcGBB . . . . .	81
NegLLMultiBin . . . . .	82
NegLLTriBin . . . . .	83
pAddBin . . . . .	84
pBETA . . . . .	85
pBetaBin . . . . .	87
pCorrBin . . . . .	89
pGBeta1 . . . . .	92
pGHGBB . . . . .	94
pGHGBeta . . . . .	96
pKUM . . . . .	98
pKumBin . . . . .	100
Plant_Disease_data . . . . .	102
pMcGBB . . . . .	103
pMultiBin . . . . .	105
pTRI . . . . .	106
pTriBin . . . . .	109
pUNI . . . . .	111
pUniBin . . . . .	112
Terror_data_ARG . . . . .	114
Terror_data_USA . . . . .	115

Alcohol\_data

*Alcohol data***Description**

Lemmens , Knibbe and Tan(1988) described a study of self reported alcohol frequencies. The no of alcohol consumption data in two reference weeks is separately self reported by a randomly selected sample of 399 respondents in the netherlands in 1983. Number of days a given individual consumes alcohol out of 7 days a week can be treated as a binomial variable. The collection of all such variables from all respondents would be defined as "binomial outcome data".

**Usage**

Alcohol\_data

**Format**

A data frame with 3 variables and 8 observations

No.D.D No of Days Drunk

Obs.fre.1 Observed frequencies for week1

Obs.fre.2 Observed frequencies for week2

**Source**

Extracted from

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. International Journal of Statistics and Probability, 2(2), pp.24-41.

Available at: <http://www.ccsenet.org/journal/index.php/ijsp/article/view/23491>

**Examples**

```
Alcohol_data$Days      # extracting the binomial random variables
sum(Alcohol_data$week2) # summing all the frequencies in week2
```

BODextract

*Binomial Data Extraction from Raw data***Description**

The below function has the ability to extract from the raw data to Binomial Outcome Data. This function simplifies the data into more presentable way to the user.

**Usage**

BODextract(data)

**Arguments**

data	vector of observations
------	------------------------

**Details**

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

**Value**

The output of BODextract gives a list format consisting  
 RV binomial random variables in vector form  
 Freq corresponding frequencies in vector form

**Examples**

```
datapoints=sample(0:10,340,replace=TRUE) #creating a sample set of observations
BODextract(datapoints) #extracting binomial outcome data from observations
Random.variable=BODextract(datapoints)$RV #extracting the binomial random variables
```

---

Chromosome_data	<i>Chromosome Data</i>
-----------------	------------------------

---

**Description**

Data in this example refer to 337 observations on the secondary association of chromosomes in Brassica; n , which is now the number of chromosomes, equals 3 and X is the number of pairs of bivalents showing association.

**Usage**

Chromosome\_data

**Format**

A data frame with 2 variables and 4 observations  
 No.of.Acco No of Associations  
 fre Observed frequencies

**Source**

Extracted from  
 Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.  
 Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990> .

**Examples**

```
Chromosome_data$No.of.Acco #extracting the binomial random variables
sum(Chromosome_data$fre) #summing all the frequencies
```

---

**Course\_data***Course Data*

---

**Description**

The data refer to the numbers of coursesaken by a class of 65 students from the first year of the Department of Statistics of Athens University of Economics. The students enrolled in this class attended 8 courses during the first year of their study. The total numbers of successful examinations (including resits) were recorded.

**Usage**

```
Course_data
```

**Format**

A data frame with 2 variables and 9 observations

sub.pass	subjects passed
fre	Observed frequencies

**Source**

Extracted from

Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In Advances in Mathematical and Statistical Modeling. Boston: Birkhäuser Boston, pp. 21-33.

Available at: [http://dx.doi.org/10.1007/978-0-8176-4626-4\\_2](http://dx.doi.org/10.1007/978-0-8176-4626-4_2).

**Examples**

```
Course_data$sub.pass      # extracting the binomial random variables
sum(Course_data$fre)      # summing all the frequencies
```

---

**dAddBin***Additive Binomial Distribution*

---

**Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Additive Binomial Distribution.

**Usage**

```
dAddBin(x,n,p,alpha)
pAddBin(x,n,p,alpha)
```

### Arguments

x	vector of binomial random variables
n	single value for no of binomial trials
p	single value for probability of success
alpha	single value for alpha parameter

### Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values

$$P_{AddBin}(x) = \binom{n}{x} p^x (1-p)^{n-x} \left( \frac{\text{alpha}}{2} \left( \frac{x(x-1)}{p} + \frac{(n-x)(n-x-1)}{(1-p)} - \frac{\text{alpha}(n-1)}{2} \right) + 1 \right)$$

$$x = 0, 1, 2, 3, \dots n$$

$$n = 1, 2, 3, \dots$$

$$0 < p < 1$$

$$-1 < \text{alpha} < 1$$

The mean and the variance are denoted as

$$E_{AddBin}[x] = np$$

$$Var_{AddBin}[x] = np(1-p)(1 + (n-1)\text{alpha})$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of dAddBin gives a list format consisting

pdf probability function values in vector form

mean mean of Additive Binomial Distribution

var variance of Additive Binomial Distribution

The output of pAddBin gives cumulative probability values in vector form.

### References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.

Available at: <http://www.jstor.org/stable/2529589>.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990>.

Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

## Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(0.58,0.59,0.6,0.61,0.62)
b<-c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Additive binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dAddBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dAddBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}
dAddBin(0:10,10,0.58,0.022)$pdf      #extracting the probability values
dAddBin(0:10,10,0.58,0.022)$mean    #extracting the mean
dAddBin(0:10,10,0.58,0.022)$var     #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(5)
a<-c(0.58,0.59,0.6,0.61,0.62)
b<-c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Additive binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
  lines(0:10,pAddBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
  points(0:10,pAddBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}
pAddBin(0:10,10,0.58,0.022)      #acquiring the cumulative probability values
```

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Beta Distribution bounded between [0,1]

## Usage

```
dBETA(p,a,b)
pBETA(p,a,b)
mazBETA(r=1,a,b)
```

## Arguments

p	vector of probabilities
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
r	vector of moments

### Details

The probability density function and cumulative density function of a unit bounded beta distribution with random variable P are given by

$$g_P(p) = \frac{p^{a-1}(1-p)^{b-1}}{B(a, b)}$$

;  $0 \leq p \leq 1$

$$G_P(p) = \frac{B_p(a, b)}{B(a, b)}$$

;  $0 \leq p \leq 1$

$a, b > 0$

The mean and the variance are denoted by

$$E[P] = \frac{a}{a+b}$$

$$var[P] = \frac{ab}{(a+b)^2(a+b+1)}$$

The moments about zero is denoted as

$$E[P^r] = \prod_{i=0}^{r-1} \left( \frac{a+i}{a+b+i} \right)$$

$r = 1, 2, 3, \dots$

Defined as  $B_p(a, b) = \int_0^p t^{a-1}(1-t)^{b-1} dt$  is incomplete beta integrals and  $B(a, b)$  is the beta function.

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of dBETA gives a list format consisting

pdf probability density values in vector form

mean mean of the beta distribution

var variance of the beta distribution

The output of pBETA gives the cumulative density values in vector form.

The output of mazBETA gives the moments about zero in vector form.

### References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley

Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.

Available at: <http://linkinghub.elsevier.com/retrieve/pii/0167947396900158>.

**See Also**[Beta](#)

or

<https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Beta.html>**Examples**

```
#plotting the random variables and probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),dBETA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dBETA(seq(0,1,by=0.01),2,3)$pdf  #extracting the pdf values
dBETA(seq(0,1,by=0.01),2,3)$mean #extracting the mean
dBETA(seq(0,1,by=0.01),2,3)$var  #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),pBETA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pBETA(seq(0,1,by=0.01),2,3)  #acquiring the cumulative probability values
mazBETA(1.4,3,2)            #acquiring the moment about zero values
mazBETA(2,3,2)-mazBETA(1,3,2)^2 #acquiring the variance for a=3,b=2
mazBETA(1.9,5.5,6)          #only the integer value of moments is taken here because moments cannot be decimal
```

**Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Beta-Binomial Distribution.

**Usage**

```
dBetaBin(x,n,a,b)
pBetaBin(x,n,a,b)
```

### Arguments

x	vector of binomial random variables
n	single value for no of binomial trials
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b

### Details

Mixing beta distribution with binomial distribution will create the Beta-Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values

$$P_{BetaBin}(x) = \binom{n}{x} \frac{B(a+x, n+b-x)}{B(a, b)}$$

$$a, b > 0$$

$$x = 0, 1, 2, 3, \dots, n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{BetaBin}[x] = \frac{na}{a+b}$$

$$Var_{BetaBin}[x] = \frac{(nab)}{(a+b)^2} \frac{(a+b+n)}{(a+b+1)}$$

$$overdispersion = \frac{1}{a+b+1}$$

Defined as  $B(a, b)$  is the beta function.

### Value

The output of dBetaBin gives a list format consisting

pdf probability function values in vector form

mean mean of the Beta-Binomial Distribution

var variance of the Beta-Binomial Distribution

over.dis.para over dispersion value of the Beta-Binomial Distribution

The output of pBetaBin gives cumulative probability values in vector form.

### References

Young-Xu, Y. & Chan, K.A., 2008. Pooling overdispersed binomial data to estimate event rate. BMC medical research methodology, 8(1), p.58.

Available at: <http://www.ncbi.nlm.nih.gov/pmc/articles/PMC2538541/>&tool=pmcentrez&rendertype=abstract .

Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.

Available at: <http://linkinghub.elsevier.com/retrieve/pii/0167947396900158> .

Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. *Phytopathology*, 83(9), p.759.

Available at: [http://www.apsnet.org/publications/phytopathology/backissues/Documents/1993Abstracts/Phyto\\_83\\_759.htm](http://www.apsnet.org/publications/phytopathology/backissues/Documents/1993Abstracts/Phyto_83_759.htm)

## Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(1,2,5,10,0.2)
plot(0,0,main="Beta-binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dBetaBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dBetaBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}

dBetaBin(0:10,10,4,.2)$pdf    #extracting the pdf values
dBetaBin(0:10,10,4,.2)$mean   #extracting the mean
dBetaBin(0:10,10,4,.2)$var    #extracting the variance
dBetaBin(0:10,10,4,.2)$over.dis.para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
  lines(0:10,pBetaBin(0:10,10,a[i],a[i]),col = col[i])
  points(0:10,pBetaBin(0:10,10,a[i],a[i]),col = col[i])
}
pBetaBin(0:10,10,4,.2)    #acquiring the cumulative probability values
```

dCorrBin

*Correlated Binomial Distribution*

## Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Correlated Binomial Distribution.

## Usage

```
dCorrBin(x,n,p,cov)
pCorrBin(x,n,p,cov)
```

## Arguments

x	vector of binomial random variables
n	single value for no of binomial trials

p	single value for probability of success
cov	single value for covariance

### Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values

$$P_{Corrbin}(x) = \binom{n}{x} (p^x)(1-p)^{n-x} \left(1 + \left(\frac{cov}{2p^2(1-p)^2}\right)((x-np)^2 + x(2p-1) - np^2)\right)$$

$$x = 0, 1, 2, 3, \dots n$$

$$n = 1, 2, 3, \dots$$

$$0 < p < 1$$

$$-\infty < cov < +\infty$$

The Correlation is inbetween

$$\frac{-2}{n(n-1)} \min\left(\frac{p}{1-p}, \frac{p}{1-p}\right) \leq correlation \leq \frac{2p(1-p)}{(n-1)p(1-p) + 0.25 - fo}$$

$$where fo = \min[(x - (n-1)p - 0.5)^2]$$

The mean and the variance are denoted as

$$E_{Corrbin}[x] = np$$

$$Var_{Corrbin}[x] = np(1-p)(1+(n-1)cov)$$

$$Corr_{Corrbin}[x] = \frac{cov}{p(1-p)}$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of dCorrBin gives a list format consisting

pdf probability function values in vector form

mean mean of Correlated Binomial Distribution

var variance of Correlated Binomial Distribution

corr correlation of Correlated Binomial Distribution

mincorr minimum correlation value possible

maxcorr maximum correlation value possible

The output of pCorrBin gives cumulative probability values in vector form.

## References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.

Available at: <http://www.jstor.org/stable/2529589>.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.

Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990>.

Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

## See Also

[CBprob](#)

or

<https://cran.r-project.org/web/packages/BinaryEPPM/BinaryEPPM.pdf>

## Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(0.58,0.59,0.6,0.61,0.62)
b<-c(0.022,0.023,0.024,0.025,0.026)
plot(0:0,main="Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dCorrBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dCorrBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}
dCorrBin(0:10,10,0.58,0.022)$pdf      #extracting the pdf values
dCorrBin(0:10,10,0.58,0.022)$mean    #extracting the mean
dCorrBin(0:10,10,0.58,0.022)$var     #extracting the variance
dCorrBin(0:10,10,0.58,0.022)$corr    #extracting the correlation
dCorrBin(0:10,10,0.58,0.022)$mincorr #extracting the minimum correlation value
dCorrBin(0:10,10,0.58,0.022)$maxcorr #extracting the maximum correlation value

#plotting the random variables and cumulative probability values
col<-rainbow(5)
a<-c(0.58,0.59,0.6,0.61,0.62)
b<-c(0.022,0.023,0.024,0.025,0.026)
plot(0:0,main="Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
  lines(0:10,pCorrBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
  points(0:10,pCorrBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}
pCorrBin(0:10,10,0.58,0.022)      #acquiring the cumulative probability values
```

dGBeta1

*Generalized Beta Type-1 Distribution***Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Generalized Beta Type-1 Distribution bounded between [0,1]

**Usage**

```
dGBeta1(p,a,b,c)
pGBeta1(p,a,b,c)
mazGBeta1(r=1,a,b,c)
```

**Arguments**

p	vector of probabilities
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
c	single value for shape parameter gamma representing as c
r	vector of moments

**Details**

The probability density function and cumulative density function of a unit bounded Generalized Beta Type-1 Distribution with random variable P are given by

$$g_P(p) = \frac{c}{B(a, b)} p^{ac-1} (1 - p^c)^{b-1}$$

;  $0 \leq p \leq 1$

$$G_P(p) = \frac{p^{ac}}{aB(a, b)} 2F1(a, 1 - b; p^c; a + 1)$$

$0 \leq p \leq 1$

$$a, b, c > 0$$

The mean and the variance are denoted by

$$E[P] = \frac{B(a + b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$var[P] = \frac{B(a + b, \frac{2}{c})}{B(a, \frac{2}{c})} - \left( \frac{B(a + b, \frac{1}{c})}{B(a, \frac{1}{c})} \right)^2$$

The moments about zero is denoted as

$$E[P^r] = \frac{B(a + b, \frac{r}{c})}{B(a, \frac{r}{c})}$$

$r = 1, 2, 3, \dots$

Defined as  $B(a, b)$  is beta function Defined as  $2F1(a, b; c; d)$  is Gaussian Hypergeometric function

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of dGBeta1 gives a list format consisting  
pdf probability density values in vector form  
mean mean of the Generalized Beta Type-1 Distribution  
var variance of the Generalized Beta Type-1 Distribution  
The output pGBeta1 gives the cumulative density values in vector form.  
The output mazGBeta1 gives the moments about zero in vector form.

### References

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. International Journal of Statistics and Probability, 2(2), pp.24-41.

Available at: <http://www.ccsenet.org/journal/index.php/ijsp/article/view/23491> .

Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of McDonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.

Roozegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. Communications in Statistics - Simulation and Computation, (May), pp.0-0.

Available at: <http://www.tandfonline.com/doi/full/10.1080/03610918.2015.1088024> .

### Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
  lines(seq(0,1,by=0.001),dGBeta1(seq(0,1,by=0.001),a[i],1,2*a[i])$pdf,col = col[i])
}
dGBeta1(seq(0,1,by=0.01),2,3,1)$pdf    #extracting the pdf values
dGBeta1(seq(0,1,by=0.01),2,3,1)$mean   #extracting the mean
dGBeta1(seq(0,1,by=0.01),2,3,1)$var    #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(5)
a<-c(.001,.002,.03,1.5,2.15)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:5)
{
  lines(seq(0,1,by=0.001),pGBeta1(seq(0,1,by=0.001),a[i],1,12+a[i]),col=col[i])
}

pGBeta1(seq(0,1,by=0.01),2,3,1)    #acquiring the cumulative probability values
mazGBeta1(1.4,3,2,2)                #acquiring the moment about zero values
mazGBeta1(2,3,2,2)-mazGBeta1(1,3,2,2)^2      #acquiring the variance for a=3,b=2,c=2
mazGBeta1(3.2,3,2,2)      #only the integer value of moments is taken here because moments cannot be decimal
```

dGHGBB

*Gaussian Hypergeometric Generalized Beta Binomial Distribution*

## Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Gaussian Hypergeometric Generalized Beta Binomial distribution.

## Usage

```
dGHGBB(x,n,a,b,c)
pGHGBB(x,n,a,b,c)
```

## Arguments

x	vector of binomial random variables
n	single value for no of binomial trials
a	single value for shape parameter alpha value representing a
b	single value for shape parameter beta value representing b
c	single value for shape parameter lambda value representing c

## Details

Mixing Gaussian Hypergeometric Generalized Beta distribution with binomial distribution will create the Gaussian Hypergeometric Generalized Beta Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values

$$P_{GHGBB}(x) = \frac{1}{2F1(-n, a; -b - n + 1; c)} \binom{n}{x} \frac{B(x + a, n - x + b)}{B(a, b + n)} (c^x)$$

$$a, b, c > 0$$

$$x = 0, 1, 2, \dots n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{GHGBB}[x] = nE_{GHGBeta}$$

$$Var_{GHGBB}[x] = nE_{GHGBeta}(1 - E_{GHGBeta}) + n(n - 1)Var_{GHGBeta}$$

$$overdispersion = \frac{var_{GHGBeta}}{E_{GHGBeta}(1 - E_{GHGBeta})}$$

Defined as  $B(a, b)$  is the beta function. Defined as  $2F1(a, b; c; d)$  is the gaussian hypergeometric function

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dGHGBB gives a list format consisting  
 pdf probability function values in vector form  
 mean mean of Gaussian Hypergeometric Generalized Beta Binomial Distribution  
 var variance of Gaussian Hypergeometric Generalized Beta Binomial Distribution  
 over.dis.para over dispersion value of Gaussian Hypergeometric Generalized Beta Binomial Dis-  
 tribution

The output of pGHGBB gives cumulative probability function values in vector form

### References

- Rodríguez-Avi, J., Conde-Sánchez, A., Sáez-Castillo, A. J., & Olmo-Jiménez, M. J. (2007). A generalization of the beta-binomial distribution. Journal of the Royal Statistical Society. Series C (Applied Statistics), 56(1), 51-61.
- Available at : <http://dx.doi.org/10.1111/j.1467-9876.2007.00564.x>
- Pearson, J., 2009. Computation of Hypergeometric Functions. Transformation, (September), p.1–123.

### See Also

[hypergeo\\_powerseries](#)  
 or  
<https://cran.r-project.org/web/packages/hypergeo/hypergeo.pdf>

### Examples

```
#plotting the random variables and probability values
col<-rainbow(6)
a<-c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="GHGBB probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,7),ylim = c(0,0.9))
for (i in 1:6)
{
  lines(0:7,dGHGBB(0:7,7,1+a[i],0.3,1+a[i])$pdf,col = col[i],lwd=2.85)
  points(0:7,dGHGBB(0:7,7,1+a[i],0.3,1+a[i])$pdf,col = col[i],pch=16)
}
dGHGBB(0:7,7,1.3,0.3,1.3)$pdf      #extracting the pdf values
dGHGBB(0:7,7,1.3,0.3,1.3)$mean    #extracting the mean
dGHGBB(0:7,7,1.3,0.3,1.3)$var     #extracting the variance
dGHGBB(0:7,7,1.3,0.3,1.3)$over.dis.par #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,7),ylim = c(0,1))
for (i in 1:4)
{
  lines(0:7,pGHGBB(0:7,7,1+a[i],0.3,1+a[i]),col = col[i])
  points(0:7,pGHGBB(0:7,7,1+a[i],0.3,1+a[i]),col = col[i])
}
```

```
pGHGBB(0:7,7,1.3,0.3,1.3)      #acquiring the cumulative probability values
```

dGHGBeta

*Gaussian Hypergeometric Generalized Beta Distribution*

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Gaussian Hypergeometric Generalized Beta distribution bounded between [0,1]

## Usage

```
dGHGBeta(p,n,a,b,c)
pGHGBeta(p,n,a,b,c)
mazGHGBeta(r=1,n,a,b,c)
```

## Arguments

p	vector of probabilities
n	single value for no of binomial trials
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
c	single value for shape parameter lambda representing as c
r	vector of moments

## Details

The probability density function and cumulative density function of a unit bounded Gaussian Hypergeometric Generalized Beta Distribution with random variable P are given by

$$g_P(p) = \frac{1}{B(a, b)} \frac{2F1(-n, a; -b - n + 1; 1)}{2F1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}}$$

$$; 0 \leq p \leq 1$$

$$G_P(p) = \int_0^p \frac{1}{B(a, b)} \frac{2F1(-n, a; -b - n + 1; 1)}{2F1(-n, a; -b - n + 1; c)} t^{a-1} (1-t)^{b-1} \frac{c^{b+n}}{(c + (1-c)t)^{a+b+n}} dt$$

$$; 0 \leq p \leq 1$$

$$a, b, c > 0$$

$$n = 1, 2, 3, \dots$$

The mean and the variance are denoted by

$$E[P] = \int_0^1 \frac{p}{B(a, b)} \frac{2F1(-n, a; -b - n + 1; 1)}{2F1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp$$

$$var[P] = \int_0^1 \frac{p^2}{B(a, b)} \frac{2F1(-n, a; -b - n + 1; 1)}{2F1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp - (E[p])^2$$

The moments about zero is denoted as

$$E[P^r] = \int_0^1 \frac{p^r}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}} dp$$

$r = 1, 2, 3, \dots$

Defined as  $B(a,b)$  as the beta function Defined as  $2F1(a,b;c;d)$  as the Gaussian Hypergeometric function

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dGHGBeta gives a list format consisting

pdf probability density values in vector form

mean mean of the Gaussian Hypergeometric Generalized Beta Distribution

var variance of the Gaussian Hypergeometric Generalized Beta Distribution

The output of pGHGBeta gives the cumulative density values in vector form.

The output of mazGHGBeta give the moments about zero in vector form.

### References

Rodríguez-Avi, J., Conde-Sánchez, A., Sáez-Castillo, A. J., & Olmo-Jiménez, M. J. (2007). A generalization of the beta-binomial distribution. Journal of the Royal Statistical Society. Series C (Applied Statistics), 56(1), 51-61.

Available at : <http://dx.doi.org/10.1111/j.1467-9876.2007.00564.x>

Pearson, J., 2009. Computation of Hypergeometric Functions. Transformation, (September), p.1–123.

### See Also

[hypergeo\\_powerseries](#)

or

<https://cran.r-project.org/web/packages/hypergeo/hypergeo.pdf>

### Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
  lines(seq(0,1,by=0.001),dGHGBeta(seq(0,1,by=0.001),7,1+a[i],0.3,1+a[i])$pdf,col = col[i])
}
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$pdf #extracting the pdf values
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$mean #extracting the mean
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$var #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(6)
```

```

a<-c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:6)
{
  lines(seq(0.01,1,by=0.001),pGHGBeta(seq(0.01,1,by=0.001),7,1+a[i],0.3,1+a[i]),col=col[i])
}

pGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659) #acquiring the cumulative probability values
mazGHGBeta(1.4,7,1.6312,0.3913,0.6659)           #acquiring the moment about zero values

#acquiring the variance for a=1.6312,b=0.3913,c=0.6659
mazGHGBeta(2,7,1.6312,0.3913,0.6659)-mazGHGBeta(1,7,1.6312,0.3913,0.6659)^2
mazGHGBeta(1.9,15,5,6,1)  #only the integer value of moments is taken here because moments cannot be decimal

```

---

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Kumaraswamy Distribution bounded between [0,1]

## Usage

```
dKUM(p,a,b)
pKUM(p,a,b)
mazKUM(r=1,a,b)
```

## Arguments

p	vector of probabilities
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
r	vector of moments

## Details

The probability density function and cumulative density function of a unit bounded Kumaraswamy Distribution with random variable P are given by

$$g_P(p) = abp^{a-1}(1-p^a)^{b-1}$$

$$; 0 \leq p \leq 1$$

$$G_P(p) = 1 - (1-p^a)^b$$

$$; 0 \leq p \leq 1$$

$$a, b > 0$$

The mean and the variance are denoted by

$$E[P] = bB\left(1 + \frac{1}{a}, b\right)$$

$$var[P] = bB\left(1 + \frac{2}{a}, b\right) - \left(bB\left(1 + \frac{1}{a}, b\right)\right)^2$$

The moments about zero is denoted as

$$E[P^r] = bB\left(1 + \frac{r}{a}, b\right)$$

$r = 1, 2, 3, \dots$

Defined as  $B(a, b)$  is the beta function.

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of `dKUM` gives a list format consisting  
`pdf` probability density values in vector form  
`mean` mean of the kumaraswamy distribution  
`var` variance of the kumaraswamy distribution  
The output of `pKUM` gives the cumulative density values in vector form.  
The output of `mazKUM` gives the moments about zero in vector form.

### References

- Kumaraswamy, P. (1980). A generalized probability density function for double-bounded random processes. *Journal of Hydrology*, 46(1), 79-88.  
Available at : [http://dx.doi.org/10.1016/0022-1694\(80\)90036-0](http://dx.doi.org/10.1016/0022-1694(80)90036-0)
- Jones, M. C. (2009). Kumaraswamy's distribution: A beta-type distribution with some tractability advantages. *Statistical Methodology*, 6(1), 70-81.  
Available at : <http://dx.doi.org/10.1016/j.stamet.2008.04.001>

### See Also

[Kumaraswamy](#)

or

<https://cran.r-project.org/web/packages/extrDistr/extrDistr.pdf>

### Examples

```
#plotting the random variables and probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,6))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),dKUM(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}
```

```

dKUM(seq(0,1,by=0.01),2,3)$pdf    #extracting the probability values
dKUM(seq(0,1,by=0.01),2,3)$mean   #extracting the mean
dKUM(seq(0,1,by=0.01),2,3)$var    #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),pKUM(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}
pKUM(seq(0,1,by=0.01),2,3)      #acquiring the cumulative probability values
mazKUM(1.4,3,2)                #acquiring the moment about zero values
mazKUM(2,2,3)-mazKUM(1,2,3)^2  #acquiring the variace for a=2,b=3
mazKUM(1.9,5.5,6)              #only the integer value of moments is taken here because moments cannot be decimal

```

**dKumBin***Kumaraswamy Binomial Distribution***Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Kumaraswamy Binomial Distribution.

**Usage**

```
dKumBin(x,n,a,b,it=25000)
pKumBin(x,n,a,b,it=25000)
```

**Arguments**

x	vector of binomial random variables
n	single value for no of binomial trial
a	single value for shape parameter alpha representing a
b	single value for shape parameter beta representing b
it	number of iterations to converge as a proper probability function replacing infinity

**Details**

Mixing kumaraswamy distribution with binomial distribution will create the Kumaraswamy Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values

$$P_{KumBin}(x) = ab \binom{n}{x} \sum_{j=0}^{it} (-1)^j \binom{b-1}{j} B(x + a + aj, n - x + 1)$$

$$\begin{aligned} a, b &> 0 \\ x &= 0, 1, 2, \dots n \\ n &= 1, 2, 3, \dots \\ it &> 0 \end{aligned}$$

The mean, variance and over dispersion are denoted as

$$\begin{aligned} E_{KumBin}[x] &= nbB\left(1 + \frac{1}{a}, b\right) \\ Var_{KumBin}[x] &= n^2b\left(B\left(1 + \frac{2}{a}, b\right) - bB\left(1 + \frac{1}{a}, b\right)^2\right) + nb\left(B\left(1 + \frac{1}{a}, b\right) - B\left(1 + \frac{2}{a}, b\right)\right) \\ overdispersion &= \frac{\left(bB\left(1 + \frac{2}{a}, b\right) - \left(bB\left(1 + \frac{1}{a}, b\right)\right)^2\right)}{\left(bB\left(1 + \frac{1}{a}, b\right) - \left(bB\left(1 + \frac{1}{a}, b\right)\right)^2\right)} \end{aligned}$$

Defined as  $B(a, b)$  is the beta function

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of dKumBin gives a list format consisting  
 pdf probability function values in vector form  
 mean mean of the Kumaraswamy Binomial Distribution  
 var variance of the Kumaraswamy Binomial Distribution  
 over.dis.para over dispersion value of the Kumaraswamy Distribution  
 The output of pKumBin gives cumulative probability values in vector form.

### References

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

### Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(1,2,5,10,.85)
plot(0,0,main="Kumaraswamy binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dKumBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dKumBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}
dKumBin(0:10,10,4,2)$pdf #extracting the pdf values
dKumBin(0:10,10,4,2)$mean #extracting the mean
dKumBin(0:10,10,4,2)$var #extracting the variance
dKumBin(0:10,10,4,2)$over.dis.para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col<-rainbow(5)
```

```
a<-c(1,2,5,10,.85)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
  lines(0:10,pKumBin(0:10,10,a[i],a[i]),col = col[i])
  points(0:10,pKumBin(0:10,10,a[i],a[i]),col = col[i])
}
pKumBin(0:10,10,4,2)      #acquiring the cumulative probability values
```

---

## Description

These functions provide the ability for generating probability function values and cumulative probability function values for the McDonald Generalized Beta Binomial Distribution.

## Usage

```
dMcGBB(x,n,a,b,c)
pMcGBB(x,n,a,b,c)
```

## Arguments

x	vector of binomial random variables
n	single value for no of binomial trials
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
c	single value for shape parameter gamma representing as c

## Details

Mixing Generalized Beta Type-1 Distribution with binomial distribution the probability function value and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values

$$P_{McGBB}(x) = \binom{n}{x} \frac{1}{B(a,b)} \left( \sum_{j=0}^{n-x} (-1)^j \binom{n-x}{j} B\left(\frac{x}{c} + a + \frac{j}{c}, b\right) \right)$$

$$a, b, c > 0$$

The mean, variance and over dispersion are denoted as

$$E_{McGBB}[x] = n \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$Var_{McGBB}[x] = n^2 \left( \frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - \left( \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} \right)^2 \right) + n \left( \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} - \frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} \right)$$

$$\text{overdispersion} = \frac{\frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - \left(\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})}\right)^2}{\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} - \left(\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})}\right)^2}$$

$$x = 0, 1, 2, \dots n$$

$$n = 1, 2, 3, \dots$$

### Value

The output of dMcGBB gives a list format consisting  
 pdf probability function values in vector form  
 mean mean of McDonald Generalized Beta Binomial Distribution  
 var variance of McDonald Generalized Beta Binomial Distribution  
 over.dis.para over dispersion value of McDonald Generalized Beta Binomial Distribution  
 The output of pMcGBB gives cumulative probability function values in vector form

### References

- Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. International Journal of Statistics and Probability, 2(2), pp.24-41.  
 Available at: <http://www.ccsenet.org/journal/index.php/ijsp/article/view/23491>.
- Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of McDonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.
- Roozegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. Communications in Statistics - Simulation and Computation, (May), pp.0-0.  
 Available at: <http://www.tandfonline.com/doi/full/10.1080/03610918.2015.1088024>.

### Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(1,2,5,10,0.6)
plot(0,0,main="McDonald generalized beta-binomial probability function graph",
  xlab="Binomial random variable",ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dMcGBB(0:10,10,a[i],2.5,a[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dMcGBB(0:10,10,a[i],2.5,a[i])$pdf,col = col[i],pch=16)
}
dMcGBB(0:10,10,4,2,1)$pdf          #extracting the pdf values
dMcGBB(0:10,10,4,2,1)$mean        #extracting the mean
dMcGBB(0:10,10,4,2,1)$var         #extracting the variance
dMcGBB(0:10,10,4,2,1)$over.dis.para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
  ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
```

```
{
  lines(0:10,pMcGBB(0:10,10,a[i],a[i],2),col = col[i])
  points(0:10,pMcGBB(0:10,10,a[i],a[i],2),col = col[i])
}
pMcGBB(0:10,10,4,2,1)      #acquiring the cumulative probability values
```

---

**dMultiBin***Multiplicative Binomial Distribution***Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Multiplicative Binomial Distribution.

**Usage**

```
dMultiBin(x,n,p,theta)
pMultiBin(x,n,p,theta)
```

**Arguments**

x	vector of binomial random variables
n	single value for no of binomial trials
p	single value for probability of success
theta	single value for theta

**Details**

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values

$$P_{MultiBin}(x) = \binom{n}{x} p^x (1-p)^{n-x} \frac{(theta^{x(n-x)})}{f(p,theta,n)}$$

here  $f(p,theta,n)$  is

$$f(p,theta,n) = \sum_{k=0}^n \binom{n}{k} p^k (1-p)^{n-k} (theta^{k(n-k)})$$

$$x = 0, 1, 2, 3, \dots n$$

$$n = 1, 2, 3, \dots$$

$$k = 0, 1, 2, \dots, n$$

$$0 < p < 1$$

$$0 < theta$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of `dMultiBin` gives a list format consisting  
`pdf` probability function values in vector form  
The output of `pMultiBin` gives cumulative probability values in vector form.

### References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.

Available at: <http://www.jstor.org/stable/2529589>.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.

Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990>.

### Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(0.58,0.59,0.6,0.61,0.62)
b<-c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Multiplicative binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dMultiBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dMultiBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],pch=16)
}
dMultiBin(0:10,10,.58,10.022)$pdf    #extracting the pdf values

#plotting random variables and cumulative probability values
col<-rainbow(5)
a<-c(0.58,0.59,0.6,0.61,0.62)
b<-c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Multiplicative binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
  lines(0:10,pMultiBin(0:10,10,a[i],1+b[i]),col = col[i],lwd=2.85)
  points(0:10,pMultiBin(0:10,10,a[i],1+b[i]),col = col[i],pch=16)
}
pMultiBin(0:10,10,.58,10.022)      #acquiring the cumulative probability values
```

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Triangular Distribution bounded between [0,1]

## Usage

```
dTRI(p, mode)
pTRI(p, mode)
mazTRI(r=1, mode)
```

## Arguments

p	vector of probabilities
mode	single value for mode
r	vector of moments

## Details

Setting  $\min = 0$  and  $\max = 1$   $\text{mode} = c$  in the triangular distribution a unit bounded triangular distribution can be obtained. The probability density function and cumulative density function of a unit bounded triangular distribution with random variable P are given by

$$g_P(p) = \frac{2p}{c}$$

$$; 0 \leq p < c$$

$$g_P(p) = \frac{2(1-p)}{(1-c)}$$

$$; c \leq p \leq 1$$

$$G_P(p) = \frac{p^2}{c}$$

$$; 0 \leq p < c$$

$$G_P(p) = 1 - \frac{(1-p)^2}{(1-c)}$$

$$; c \leq p \leq 1$$

$$0 \leq mode = c \leq 1$$

The mean and the variance are denoted by

$$E[P] = \frac{(a+b+c)}{3} = \frac{(1+c)}{3}$$

$$var[P] = \frac{a^2 + b^2 + c^2 - ab - ac - bc}{18} = \frac{(1+c^2-c)}{18}$$

Moments about zero is denoted as

$$E[P^r] = \frac{2c^{r+2}}{c(r+2)} + \frac{2(1-c^{r+1})}{(1-c)(r+1)} + \frac{2(c^{r+2}-1)}{(1-c)(r+2)}$$

$$r = 1, 2, 3, \dots$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of `dTRI` gives a list format consisting  
`pdf` probability density values in vector form  
`mean` mean of the unit bounded triangular distribution  
`variance` variance of the unit bounded triangular distribution  
The output of `pTRI` gives the cumulative density values in vector form.  
The output of `mazTRI` give the moments about zero in vector form.

### References

- Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.
- Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley
- Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In Advances in Mathematical and Statistical Modeling. Boston: Birkhäuser Boston, pp. 21-33.  
Available at: [http://dx.doi.org/10.1007/978-0-8176-4626-4\\_2](http://dx.doi.org/10.1007/978-0-8176-4626-4_2).
- Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. British Journal of Mathematics & Computer Science, 4(24), pp.3497-3507.  
Available at: <http://www.sciencedomain.org/abstract.php?iid=699&id=6&aid=6427>.

### See Also

[triangle](#)

or

<https://cran.r-project.org/web/packages/triangle/triangle.pdf>

---

[Triangular](#)

or

<https://cran.r-project.org/web/packages/extrDistr/extrDistr.pdf>

### Examples

```
#plotting the random variables and probability values
col<-rainbow(4)
x<-seq(0.2,0.8,by=0.2)
plot(0,0,main="Probability density graph",xlab="Random variable",
ylab="Probability density values",xlim = c(0,1),ylim = c(0,3))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),dTRI(seq(0,1,by=0.01),x[i])$pdf,col = col[i])
}

dTRI(seq(0,1,by=0.05),0.3)$pdf      #extracting the pdf values
dTRI(seq(0,1,by=0.01),0.3)$mean    #extracting the mean
dTRI(seq(0,1,by=0.01),0.3)$var     #extracting the variance

#plotting the random variables and cumulative probability values
```

```

col<-rainbow(4)
x<-seq(0.2,0.8,by=0.2)
plot(0,0,main="Cumulative density graph",xlab="Random variable",
ylab="Cumulative density values",xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pTRI(seq(0,1,by=0.01),x[i]),col = col[i])
}

pTRI(seq(0,1,by=0.05),0.3)      #acquiring the cumulative probability values
mazTRI(1.4,.3)                  #acquiring the moment about zero values
mazTRI(2,.3)-mazTRI(1,.3)^2    #variance for when is mode 0.3
mazTRI(1.9,0.5)                 #only the integer value of moments is taken here because moments cannot be decimal

```

**dTriBin***Triangular Binomial Distribution***Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Triangular Binomial distribution.

**Usage**

```
dTriBin(x,n,mode)
pTriBin(x,n,mode)
```

**Arguments**

x	vector of binomial random variables
n	single value for no of binomial trials
mode	single value for mode

**Details**

Mixing unit bounded triangular distribution with binomial distribution will create Triangular Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values

$$\begin{aligned}
P_{TriBin}(x) &= 2 \binom{n}{x} (c^{-1} B_c(x+2, n-x+1) + (1-c)^{-1} B(x+1, n-x+2) - (1-c)^{-1} B_c(x+1, n-x+2)) \\
0 < mode &= c < 1 \\
x &= 0, 1, 2, \dots n \\
n &= 1, 2, 3\dots
\end{aligned}$$

The mean, variance and over dispersion are denoted as

$$E_{TriBin}[x] = \frac{n(1+c)}{3}$$

$$Var_{TriBin}[x] = \frac{n(n+3)}{18} - \frac{n(n-3)c(1-c)}{18}$$

$$overdispersion = \frac{(1-c+c^2)}{2(2+c-c^2)}$$

Defined as  $B_c(a, b) = \int_0^c t^{a-1}(1-t)^{b-1} dt$  is incomplete beta integrals and  $B(a, b)$  is the beta function.

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of dTriBin gives a list format consisting  
 pdf probability function values in vector form  
 mean mean of the Triangular Binomial Distribution  
 var variance of the Triangular Binomial Distribution  
 over.dis.para over dispersion value of the Triangular Binomial Distribution  
 The output of pTriBin gives cumulative probability function values in vector form.

### References

- Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.
- Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In Advances in Mathematical and Statistical Modeling. Boston: Birkhäuser Boston, pp. 21-33.  
 Available at: [http://dx.doi.org/10.1007/978-0-8176-4626-4\\_2](http://dx.doi.org/10.1007/978-0-8176-4626-4_2).
- Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. British Journal of Mathematics & Computer Science, 4(24), pp.3497-3507.  
 Available at: <http://www.sciedomain.org/abstract.php?iid=699&id=6&aid=6427>.

### Examples

```
#plotting the random variables and probability values
col<-rainbow(7)
x<-seq(0.1,0.7,by=0.1)
plot(0,0,main="Triangular binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,.3))
for (i in 1:7)
{
  lines(0:10,dTriBin(0:10,10,x[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dTriBin(0:10,10,x[i])$pdf,col = col[i],pch=16)
}

dTriBin(0:10,10,.4)$pdf      #extracting the pdf values
dTriBin(0:10,10,.4)$mean    #extracting the mean
dTriBin(0:10,10,.4)$var     #extracting the variance
dTriBin(0:10,10,.4)$over.dis.para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col<-rainbow(7)
```

```

x<-seq(0.1,0.7,by=0.1)
plot(0,0,main="Triangular binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:7)
{
  lines(0:10,pTriBin(0:10,10,x[i]),col = col[i],lwd=2.85)
  points(0:10,pTriBin(0:10,10,x[i]),col = col[i],pch=16)
}
pTriBin(0:10,10,.4)    #acquiring the cumulative probability values

```

---

**dUNI***Uniform Distribution bounded between [0,1]***Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Uniform Distribution bounded between [0,1]

**Usage**

```

dUNI(p)
pUNI(p)
mazUNI(r=1)

```

**Arguments**

<b>p</b>	vector of probabilities
<b>r</b>	vector of moments

**Details**

Setting  $a = 0$  and  $b = 1$  in the Uniform Distribution a unit bounded Uniform Distribution can be obtained. The probability density function and cumulative density function of a unit bounded Uniform Distribution with random variable P are given by

$$g_P(p) = 1$$

$$0 \leq p \leq 1$$

$$G_P(p) = p$$

$$0 \leq p \leq 1$$

The mean and the variance are denoted as

$$E[P] = \frac{1}{a+b} = 0.5$$

$$var[P] = \frac{(b-a)^2}{12} = 0.0833$$

Moments about zero is denoted as

$$E[P^r] = \frac{e^{rb} - e^{ra}}{r(b-a)} = \frac{e^r - 1}{r}$$

$$r = 1, 2, 3, \dots$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of `dUNI` gives a list format consisting  
`pdf` probability density values in vector form  
`mean` mean of unit bounded uniform distribution  
`var` variance of unit bounded uniform distribution  
The output of `pUNI` gives the cumulative density values in vector form.  
The output of `mazUNI` gives the moments about zero in vector form.

### References

- Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.
- Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley

### See Also

[Uniform](#)

or

<https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Uniform.html>

### Examples

```
#plotting the random variables and probability values
plot(seq(0,1,by=0.01),dUNI(seq(0,1,by=0.01))$pdf,type = "l",main="Probability density graph",
xlab="Random variable",ylab="Probability density values")
dUNI(seq(0,1,by=0.05))$pdf      #extract the pdf values
dUNI(seq(0,1,by=0.01))$mean    #extract the mean
dUNI(seq(0,1,by=0.01))$var     #extract the variance

#plotting the random variables and cumulative probability values
plot(seq(0,1,by=0.01),pUNI(seq(0,1,by=0.01)),type = "l",main="Cumulative density graph",
xlab="Random variable",ylab="Cumulative density values")
pUNI(seq(0,1,by=0.05))      #acquiring the cumulative probability values

mazUNI(c(1,2,3))      #acquiring the moment about zero values
mazUNI(1.9)           #only the integer value of moments is taken here because moments cannot be decimal
```

### Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Uniform Binomial Distribution.

### Usage

```
dUniBin(x,n)
pUniBin(x,n)
```

### Arguments

x	vector of binomial random variables
n	single value for no of binomial trials

### Details

Mixing unit bounded uniform distribution with binomial distribution will create the Uniform Binomial Distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values

$$P_{UniBin}(x) = \frac{1}{n+1}$$

$$n = 1, 2, \dots$$

$$x = 0, 1, 2, \dots n$$

The mean, variance and over dispersion are denoted as

$$E_{UniBin}[X] = \frac{n}{2}$$

$$Var_{UniBin}[X] = \frac{n(n+2)}{12}$$

$$overdispersion = \frac{1}{3}$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dUniBin gives a list format consisting

pdf probability function values in vector form

mean mean of the Uniform Binomial Distribution

var variance of the Uniform Binomial Distribution

ove.dis.para over dispersion value of Uniform Binomial Distribution

The output of pUniBin gives cumulative probability function values in vector form.

### References

Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. British Journal of Mathematics & Computer Science, 4(24), pp.3497-3507.

Available at: <http://www.sciencedomain.org/abstract.php?id=699&id=6&aid=6427>.

## Examples

```
#plotting the binomial random variables and probability values
plot(0:10,dUniBin(0:10,10)$pdf,type="l",main="Uniform binomial probability function graph",
xlab=" Binomial random variable",ylab="Probability function values")
points(0:10,dUniBin(0:10,10)$pdf)
dUniBin(0:300,300)$pdf #extracting the pdf values
dUniBin(0:10,10)$mean #extracting the mean
dUniBin(0:10,10)$var #extracting the variance
dUniBin(0:10,10)$over.dis.para #extracting the over dispersion

#plotting the binomial random variables and cumulative probability values
plot(0:10,pUniBin(0:10,10),type="l",main="Cumulative probability function graph",
xlab=" Binomial random variable",ylab="Cumulative probability function values")
points(0:10,pUniBin(0:10,10))

pUniBin(0:15,15) #acquiring the cumulative probability values
```

EstMGFBetaBin

*Estimating the shape parameters a and b for Beta-Binomial Distribution*

## Description

The functions will estimate the shape parameters using the maximum log likelihood method and moment generating function method for the beta-binomial distribution when the binomial random variables and corresponding frequencies are given

## Usage

```
mle2(EstMLEBetaBin,start=list(a=...,b=...),data=list(x=....,freq=....),...)
EstMGFBetaBin(x,freq)
```

## Arguments

x	vector of binomial random variables
freq	vector of frequencies
start	initial values for shape parameters a and b in a list format
data	data in the form of a list for binomial random variables x and corresponding frequencies freq
.....	other inputs inherited from mle2 function

## Details

$$a, b > 0$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

`EstMLEBetaBin` here is used as a input parameter for the `mle2` function of **bbmle** package therefore output is of class of `mle2`.

The output of `EstMGFBetaBin` will produce a list format consisting  
 a shape parameter of beta distribution representing for alpha  
 b shape parameter of beta distribution representing for beta

### References

- Young-Xu, Y. & Chan, K.A., 2008. Pooling overdispersed binomial data to estimate event rate. BMC medical research methodology, 8(1), p.58.  
 Available at: <http://www.ncbi.nlm.nih.gov/pmc/articles/PMC2538541/>
- Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.  
 Available at: <http://linkinghub.elsevier.com/retrieve/pii/0167947396900158> .
- Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. Phytopathology, 83(9), p.759.  
 Available at: [http://www.apsnet.org/publications/phytopathology/backissues/Documents/1993Abstracts/Phyto\\_83\\_759.htm](http://www.apsnet.org/publications/phytopathology/backissues/Documents/1993Abstracts/Phyto_83_759.htm)

### See Also

`mle2`  
`mle2-class`  
 or  
<https://cran.r-project.org/web/packages/bbmle/bbmle.pdf>

### Examples

```
No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies
#estimating the parameters using maximum log likelihood value and assigning it
parameters=bbmle::mle2(EstMLEBetaBin,start = list(a=0.1,b=0.1),data = list(x=No.D.D,freq=Obs.fre.1))
bbmle::coef(parameters)  #extracting the parameters

#estimating the parameters using moment generating function methods
EstMGFBetaBin(No.D.D,Obs.fre.1)
```

### Description

The function will estimate the probability of success and alpha using the maximum log likelihood method for the Additive Binomial distribution when the binomial random variables and corresponding frequencies are given

### Usage

```
EstMLEAddBin(x, freq)
```

### Arguments

x	vector of binomial random variables
freq	vector of frequencies

### Details

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of EstMLEAddBin will produce a list consisting  
 p probability of success  
 alpha alpha

### References

- Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.
- L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.  
 Available at: <http://www.jstor.org/stable/2529589>.
- Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.  
 Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990>.
- Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

### Examples

```
No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies
EstMLEAddBin(No.D.D,Obs.fre.1)           #estimating the probability value and alpha value
EstMLEAddBin(No.D.D,Obs.fre.1)$p         #extracting the estimated probability value
```

EstMLEBetaBin	<i>Estimating the shape parameters a and b for Beta-Binomial Distribution</i>
---------------	---

## Description

The functions will estimate the shape parameters using the maximum log likelihood method and moment generating function method for the beta-binomial distribution when the binomial random variables and corresponding frequencies are given

## Usage

```
mle2(EstMLEBetaBin,start=list(a=...,b=...),data=list(x=....,freq=....),...)
EstMGFBetaBin(x,freq)
```

## Arguments

x	vector of binomial random variables
freq	vector of frequencies
start	initial values for shape parameters a and b in a list format
data	data in the form of a list for binomial random variables x and corresponding frequencies freq
.....	other inputs inherited from mle2 function

## Details

$$a, b > 0$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

## Value

EstMLEBetaBin here is used as a input parameter for the `mle2` function of **bbmle** package therefore output is of class of `mle2`.

The output of EstMGFBetaBin will produce a list format consisting

a shape parameter of beta distribution representing for alpha

b shape parameter of beta distribution representing for beta

## References

- Young-Xu, Y. & Chan, K.A., 2008. Pooling overdispersed binomial data to estimate event rate. BMC medical research methodology, 8(1), p.58.  
 Available at: <http://www.ncbi.nlm.nih.gov/pmc/articles/PMC2538541/>&tool=pmcentrez&rendertype=abstract .
- Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.  
 Available at: <http://linkinghub.elsevier.com/retrieve/pii/0167947396900158> .
- Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. Phytopathology, 83(9), p.759.  
 Available at: [http://www.apsnet.org/publications/phytopathology/backissues/Documents/1993Abstracts/Phyto\\_83\\_759.htm](http://www.apsnet.org/publications/phytopathology/backissues/Documents/1993Abstracts/Phyto_83_759.htm)

## See Also

[mle2](#)

[mle2-class](#)

or

<https://cran.r-project.org/web/packages/bbmle/bbmle.pdf>

## Examples

```
No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies
#estimating the parameters using maximum log likelihood value and assigning it
parameters=bbmle::mle2(EstMLEBetaBin,start = list(a=0.1,b=0.1),data = list(x=No.D.D,freq=Obs.fre.1))
bbmle::coef(parameters)  #extracting the parameters

#estimating the parameters using moment generating function methods
EstMGFBetaBin(No.D.D,Obs.fre.1)
```

## Description

The function will estimate the probability of success and correlation using the maximum log likelihood method for the Correlated Binomial distribution when the binomial random variables and corresponding frequencies are given

## Usage

```
mle2(EstMLECorrBin,start=list(p=...,cov=...),data=list(x=....,freq=...),...)
```

### Arguments

x	vector of binomial random variables
freq	vector of frequencies
start	initial values for the probability of success p and covariance parameter cov
data	data in the form of a list for binomial random variables x and corresponding frequencies freq
.....	other inputs inherited from mle2 function

### Details

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

$$0 < p < 1$$

$$-\infty < cov < +\infty$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

EstMLECorrBin here is used as a input parameter for the mle2 function of **bbmle** package therefore output is of class of mle2.

### References

- Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.
- L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.  
Available at: <http://www.jstor.org/stable/2529589>.
- Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.  
Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990>.
- Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

### See Also

[mle2](#)

[mle2-class](#)

or

<https://cran.r-project.org/web/packages/bbmle/bbmle.pdf>

## Examples

```
No.D.D=0:7           #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters=bbmle::mle2(EstMLECorrBin,start = list(p=0.5,cov=0.0050),data = list(x=No.D.D,freq=Obs.fre.1))
bbmle::coef(parameters)          #extracting the parameters
```

---

EstMLEGHGBB

*Estimating the shape parameters a,b and c for Gaussian Hypergeometric Generalized Beta Binomial Distribution*

---

## Description

The function will estimate the shape parameters using the maximum log likelihood method for the Gaussian Hypergeometric Generalized Beta Binomial distribution when the binomial random variables and corresponding frequencies are given

## Usage

```
mle2(EstMLEGHGBB,start=list(a=...,b=...,c=...),data=list(x=...,freq=...))
```

## Arguments

x	vector of binomial random variables
freq	vector of frequencies
start	initial values for shape parameters a,b and c in a list format
data	data in the form of a list for binomial random variables x corresponding frequencies freq
...	other inputs inherited from mle2 function

## Details

$$0 < a, b, c$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

## Value

EstMLEGHGBB here is used as a input parameter for the **mle2** function of **bbmle** package

## References

Rodríguez-Avi, J., Conde-Sánchez, A., Sáez-Castillo, A. J., & Olmo-Jiménez, M. J. (2007). A generalization of the beta-binomial distribution. Journal of the Royal Statistical Society. Series C (Applied Statistics), 56(1), 51-61.

Available at : <http://dx.doi.org/10.1111/j.1467-9876.2007.00564.x>

Pearson, J., 2009. Computation of Hypergeometric Functions. Transformation, (September), p.1–123.

## See Also

[hypergeo\\_powerseries](#)

or

<https://cran.r-project.org/web/packages/hypergeo/hypergeo.pdf>

---

[mle2](#)

[mle2-class](#)

or

<https://cran.r-project.org/web/packages/bbmle/bbmle.pdf>

## Examples

```
No.D.D=0:7          #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies
#estimating the parameters using maximum log likelihood value and assigning it
parameters=bbmle::mle2(EstMLEHGGB,start = list(a=0.1,b=0.1,c=0.2),data = list(x=No.D.D,freq=Obs.fre.1))
bbmle::coef(parameters)    #extracting the parameters
```

---

EstMLEKumBin

*Estimating the shape parameters a and b and iterations for Kumaraswamy Binomial Distribution*

---

## Description

The function will estimate the shape parameters using the maximum log likelihood method for the Kumaraswamy binomial distribution when the binomial random variables and corresponding frequencies are given

## Usage

```
mle2(EstMLEKumBin,start=list(a=...,b=...,it=...),data=list(x=....,freq=....),...)
```

### Arguments

x	vector of binomial random variables
freq	vector of frequencies
start	initial values for shape parameters a and b, and converge iterations it in a list format
data	data in the form of a list for binomial random variables x corresponding frequencies freq
....	other inputs inherited from mle2 function

### Details

$$0 < a, b$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

$$it > 0$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

EstMLEKumBin here is used as a input parameter for the mle2 function of **bbmle** package therefore output is of class of mle2.

### References

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

### See Also

[mle2](#)

[mle2-class](#)

or

<https://cran.r-project.org/web/packages/bbmle/bbmle.pdf>

### Examples

```
No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies
#estimating the parameters using maximum log likelihood value and assigning it

parameters1=bbmle::mle2(EstMLEKumBin,start = list(a=10.1,b=1.1,it=1000),data = list(x=No.D.D,freq=Obs.fre.1))
bbmle::coef(parameters1)    #extracting the parameters

parameters2=bbmle::mle2(EstMLEKumBin,start = list(a=10.1,b=1.1,it=5000),data = list(x=No.D.D,freq=Obs.fre.1))
bbmle::coef(parameters2)    #extracting the parameters

parameters3=bbmle::mle2(EstMLEKumBin,start = list(a=10.1,b=1.1,it=10000),data = list(x=No.D.D,freq=Obs.fre.1))
bbmle::coef(parameters3)    #extracting the parameters
```

```
parameters4=bbmle::mle2(EstMLEKumBin,start = list(a=10.1,b=1.1,it=20000),data = list(x=No.D.D,freq=Obs.freq))
bbmle::coef(parameters4) #extracting the parameters
```

**EstMLEMcGBB**

*Estimating the shape parameters a,b and c for McDonald Generalized Beta Binomial distribution*

**Description**

The function will estimate the shape parameters using the maximum log likelihood method for the McDonald Generalized Beta Binomial distribution when the binomial random variables and corresponding frequencies are given

**Usage**

```
mle2(EstMLEGHGBB,start=list(a=...,b=...,c=...),data=list(x=...,freq=...))
```

**Arguments**

x	vector of binomial random variables
freq	vector of frequencies
start	initial values for shape parameters a,b and c in a list format
data	data in the form of a list for binomial random variables x corresponding frequencies freq
...	other inputs inherited from mle2 function

**Details**

$$0 < a, b, c$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

**Value**

EstMLEMcGBB here is used as a input parameter for the mle2 function of **bbmle** package

## References

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. International Journal of Statistics and Probability, 2(2), pp.24-41.

Available at: <http://www.ccsenet.org/journal/index.php/ijsp/article/view/23491>.

Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of McDonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.

Roozegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. Communications in Statistics - Simulation and Computation, (May), pp.0-0.

Available at: <http://www.tandfonline.com/doi/full/10.1080/03610918.2015.1088024>.

## See Also

[mle2](#)

[mle2-class](#)

or

<https://cran.r-project.org/web/packages/bbmle/bbmle.pdf>

## Examples

```
No.D.D=0:7          #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies
#estimating the parameters using maximum log likelihood value and assigning it
parameters=bbmle::mle2(EstMLEMcGBB,start = list(a=0.1,b=0.1,c=0.2),data = list(x=No.D.D,freq=Obs.fre.1))
bbmle::coef(parameters)      #extracting the parameters
```

EstMLEMultiBin

*Estimating the probability of success and theta for Multiplicative Binomial Distribution*

## Description

The function will estimate the probability of success and theta parameter using the maximum log likelihood method for the Multiplicative Binomial distribution when the binomial random variables and corresponding frequencies are given

## Usage

```
mle2(EstMLEMultiBin,start=list(p=...,theta=),data=list(x=....,freq=...),...)
```

## Arguments

x	vector of binomial random variables
freq	vector of frequencies
start	initial values for the probability of success p and theta parameter theta
data	data in the form of a list for binomial random variables x and corresponding frequencies freq
.....	other inputs inherited from mle2 function

## Details

$freq \geq 0$

$x = 0, 1, 2, ..$

$0 < p < 1$

$0 < theta$

## Value

*EstMLEMultiBin* here is used as a input parameter for the `mle2` function of **bbmle** package therefore output is of class of `mle2`.

## References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.

Available at: <http://www.jstor.org/stable/2529589> .

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.

Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990> .

## See Also

`mle2`

`mle2-class`

or

<https://cran.r-project.org/web/packages/bbmle/bbmle.pdf>

## Examples

```
No.D.D=0:7          #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies
#estimating the parameters using maximum log likelihood value and assigning it
parameters=bbmle::mle2(EstMLEMultiBin,start = list(p=0.5,theta=15),data = list(x=No.D.D,freq=Obs.fre.1))
bbmle::coef(parameters)      #extracting the parameters
```

## EstMLETriBin

*Estimating the mode value for Triangular Binomial Distribution***Description**

The function will estimate the mode value using the maximum log likelihood method for the triangular binomial distribution when the binomial random variables and corresponding frequencies are given

**Usage**

```
EstMLETriBin(x, freq)
```

**Arguments**

x	vector of binomial random variables
freq	vector of frequencies

**Details**

$$0 < mode = c < 1$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

**Value**

The output of EstMLETriBin will produce a list format consisting

NegLLTriBin Negative log likelihood value for Triangular Binomial Distribution

mode Estimated mode value

**References**

Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.

Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In Advances in Mathematical and Statistical Modeling. Boston: Birkhäuser Boston, pp. 21-33.

Available at: [http://dx.doi.org/10.1007/978-0-8176-4626-4\\_2](http://dx.doi.org/10.1007/978-0-8176-4626-4_2).

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. British Journal of Mathematics & Computer Science, 4(24), pp.3497-3507.

Available at: <http://www.sciencedomain.org/abstract.php?id=699&id=6&aid=6427>.

### Examples

```
No.D.D=0:7    #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies
EstMLETriBin(No.D.D,Obs.fre.1)$mode    #estimating the mode value and extracting the mode value
```

*Exam\_data*

*Exam Data*

### Description

In an examination, there were 9 questions set on a particular topic. Each question is marked out of a total of 20 and in assessing the final class of a candidate, particular attention is paid to the total number of questions for which he has an "alpha", i.e., at least 15 out of 20, as well as his total number of marks. His number of alpha's is a rough indication of the "quality" of his exam performance. Thus, the distribution of alpha's over the candidates is of interest. There were 209 candidates attempting questions from this section of 9 questions and a total of 326 alpha's was awarded. So we treat 9 as the "litter size", and the dichotomous response is whether or not he got an alpha on the question.

### Usage

*Exam\_data*

### Format

A data frame with 2 variables and 10 observations

No.of.alpha	No of Alphas
fre	Observed frequencies

### Source

Extracted from

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990>

### Examples

<i>Exam_data\$No.of.alpha</i>	#extracting the binomial random variables
<i>sum(Exam_data\$fre)</i>	#summing all the frequencies

---

<code>fitAddBin</code>	<i>Fitting the Additive Binomial Distribution when binomial random variable, frequency, probability of success and alpha are given</i>
------------------------	--

---

### Description

The function will fit the Additive binomial distribution when random variables, corresponding frequencies, probability of success and alpha are given. It will provide the the expected frequencies, chi-squared test statistics value, p value, and degree of freedom value so that it can be seen if this distribution fits the data.

### Usage

```
fitAddBin(x,obs.freq,p,alpha,print)
```

### Arguments

<code>x</code>	vector of binomial random variables
<code>obs.freq</code>	vector of frequencies
<code>p</code>	single value for probability of success
<code>alpha</code>	single value for alpha
<code>print</code>	logical value for print or not

### Details

$$\text{obs.freq} \geq 0$$

$$x = 0, 1, 2, \dots$$

$$0 < p < 1$$

$$-1 < \text{alpha} < 1$$

### Value

THHe output of `fitAddBin` gives a list format consisting

- bin.ran.var binomial random variables
- obs.freq corresponding observed frequencies
- exp.freq corresponding expected frequencies
- statistic chi-squared test statistics
- df degree of freedom
- p.value probability value by chi-squared test statistic

## References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.

Available at: <http://www.jstor.org/stable/2529589>.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.

Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990>.

Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

## Examples

```
No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)      #assigning the corresponding the frequencies
paddbin=EstMLEAddBin(No.D.D,Obs.fre.1)$p      #assigning the estimated probability value
alphaaddbin=EstMLEAddBin(No.D.D,Obs.fre.1)$alpha #assigning the estimated alpha value

#fitting when the random variable,frequencies,probability and alpha are given
fitAddBin(No.D.D,Obs.fre.1,paddbin,alphaaddbin)

fitAddBin(No.D.D,Obs.fre.1,paddbin,alphaaddbin,F)$exp.freq      #extracting the expected frequencies
```

### fitBetaBin

*Fitting the Beta-Binomial Distribution when binomial random variable, frequency and shape parameters a and b are given*

## Description

The function will fit the beta-binomial distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

## Usage

```
fitBetaBin(x,obs.freq,a,b,print)
```

## Arguments

x	vector of binomial random variables
obs.freq	vector of frequencies
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
print	logical value for print or not

## Details

$$0 < a, b$$

$$x = 0, 1, 2, \dots, n$$

$$obs.freq \geq 0$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

## Value

The output of `fitBetaBin` gives a list format consisting  
`bin.ran.var` binomial random variables  
`obs.freq` corresponding observed frequencies  
`exp.freq` corresponding expected frequencies  
`statistic` chi-squared test statistics  
`df` degree of freedom  
`p.value` probability value by chi-squared test statistic  
`over.dis.para` over dispersion value.

## References

- Young-Xu, Y. & Chan, K.A., 2008. Pooling overdispersed binomial data to estimate event rate. BMC medical research methodology, 8(1), p.58.  
 Available at: <http://www.ncbi.nlm.nih.gov/pmc/articles/PMC2538541/>&tool=pmcentrez&rendertype=abstract .
- Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.  
 Available at: <http://linkinghub.elsevier.com/retrieve/pii/0167947396900158> .
- Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. Phytopathology, 83(9), p.759.  
 Available at: [http://www.apsnet.org/publications/phytopathology/backissues/Documents/1993Abstracts/Phyto\\_83\\_759.htm](http://www.apsnet.org/publications/phytopathology/backissues/Documents/1993Abstracts/Phyto_83_759.htm)

## See Also

`mle2`

`mle2-class`

or

<https://cran.r-project.org/web/packages/bbmle/bbmle.pdf>

## Examples

```
No.D.D=0:7    #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies
#estimating the parameters using maximum log likelihood value and assigning it
parameters=bbmle::mle2(EstMLEBetaBin,start = list(a=0.1,b=0.1),data = list(x=No.D.D,freq=Obs.fre.1))
bbmle::coef(parameters)  #extracting the parameters a and b
aBetaBin=bbmle::coef(parameters)[1]  #assigning the parameter a
bBetaBin=bbmle::coef(parameters)[2]  #assigning the parameter b
#fitting when the random variable,frequencies,shape parameter values are given.
fitBetaBin(No.D.D,Obs.fre.1,aBetaBin,bBetaBin)

#estimating the parameters using moment generating function methods
EstMGFBetaBin(No.D.D,Obs.fre.1)
aBetaBin1=EstMGFBetaBin(No.D.D,Obs.fre.1)$a  #assigning the estimated a
bBetaBin1=EstMGFBetaBin(No.D.D,Obs.fre.1)$b  #assigning the estimated b
#fitting when the random variable,frequencies,shape parameter values are given.
fitBetaBin(No.D.D,Obs.fre.1,aBetaBin1,bBetaBin1)

fitBetaBin(No.D.D,Obs.fre.1,aBetaBin1,bBetaBin1,F)$exp.freq  #extracting the expected frequencies
```

**fitBin**

*Fitting the Binomial Distribution when binomial random variable, frequency and probability value are given*

## Description

The function will fit the binomial distribution when random variables, corresponding frequencies and probability value are given. It will provide the expected frequencies, chi-squared test statistics value, p value and degree of freedom so that it can be seen if this distribution fits the data.

## Usage

```
fitBin(x,obs.freq,p=0,print=T)
```

## Arguments

x	vector of binomial random variables
obs.freq	vector of frequencies
p	single value for probability
print	logical value for print or not

## Details

$$x = 0, 1, 2, \dots$$

$$0 \leq p \leq 1$$

$$obs.freq \geq 0$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of `fitBin` gives a list format consisting  
`bin.ran.var` binomial random variables  
`obs.freq` corresponding observed frequencies  
`exp.freq` corresponding expected frequencies  
`statistic` chi-squared test statistics value  
`df` degree of freedom  
`p.value` probability value by chi-squared test statistic

**Examples**

```
No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

#fitting when the random variable,frequencies,probability value are given.
fitBin(No.D.D,Obs.fre.1,p=0.7)

fitBin(No.D.D,Obs.fre.1,p=0.7,F)$exp.freq #extracting the expected frequencies

#fitting when the random variable,frequencies are given.
fitBin(No.D.D,Obs.fre.1)
```

**fitCorrBin**

*Fitting the Correlated Binomial Distribution when binomial random variable, frequency, probability of success and covariance are given*

**Description**

The function will fit the Correlated binomial Distribution when random variables, corresponding frequencies, probability of success and covariance are given. It will provide the the expected frequencies, chi-squared test statistics value, p value, and degree of freedom so that it can be seen if this distribution fits the data.

**Usage**

```
fitCorrBin(x,obs.freq,p,cov,print)
```

**Arguments**

<code>x</code>	vector of binomial random variables
<code>obs.freq</code>	vector of frequencies
<code>p</code>	single value for probability of success
<code>cov</code>	single value for covariance
<code>print</code>	logical value for print or not

## Details

$obs.freq \geq 0$

$x = 0, 1, 2, ..$

$0 < p < 1$

$-\infty < cov < +\infty$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

## Value

The output of fitCorrBin gives a list format consisting  
**bin.ran.var** binomial random variables  
**obs.freq** corresponding observed frequencies  
**exp.freq** corresponding expected frequencies  
**statistic** chi-squared test statistics  
**df** degree of freedom  
**p.value** probability value by chi-squared test statistic  
**corr** Correlation value

## References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.

Available at: <http://www.jstor.org/stable/2529589> .

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990> .

Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

## Examples

```
No.D.D=0:7          #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters=bbmle::mle2(EstMLECorrBin,start = list(p=0.5,cov=0.0050),data = list(x=No.D.D,freq=Obs.fre.1))
pCorrbin=bbmle::coef(parameters)[1]
covCorrbin=bbmle::coef(parameters)[2]
#fitting when the random variable,frequencies,probability and covariance are given
fitCorrBin(No.D.D,Obs.fre.1,pCorrbin,covCorrbin)

fitCorrBin(No.D.D,Obs.fre.1,pCorrbin,covCorrbin,F)$exp.freq  #extracting the expected frequencies
```

---

fitGHGBB	<i>Fitting the Gaussian Hypergeometric Generalized Beta Binomial Distribution when binomial random variable, frequency and shape parameters a,b and c are given</i>
----------	---

---

**Description**

The function will fit the Gaussian Hypergeometric Generalized Beta Binomial Distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

**Usage**

```
fitGHGBB(x,obs.freq,a,b,c,print)
```

**Arguments**

x	vector of binomial random variables
obs.freq	vector of frequencies
a	single value for shape parameter alpha representing a
b	single value for shape parameter beta representing b
c	single value for shape parameter lambda representing c
print	logical value for print or not

**Details**

$$0 < a, b, c$$

$$x = 0, 1, 2, \dots$$

$$obs.freq \geq 0$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

**Value**

The output of fitGHGBB gives a list format consisting  
**bin.ran.var** binomial random variables  
**obs.freq** corresponding observed frequencies  
**exp.freq** corresponding expected frequencies  
**statistic** chi-squared test statistics  
**df** degree of freedom  
**p.value** probability value by chi-squared test statistic  
**over.dis.para** over dispersion value.

## References

Rodríguez-Avi, J., Conde-Sánchez, A., Sáez-Castillo, A. J., & Olmo-Jiménez, M. J. (2007). A generalization of the beta-binomial distribution. Journal of the Royal Statistical Society. Series C (Applied Statistics), 56(1), 51-61.

Available at : <http://dx.doi.org/10.1111/j.1467-9876.2007.00564.x>

Pearson, J., 2009. Computation of Hypergeometric Functions. Transformation, (September), p.1–123.

## See Also

[hypergeo\\_powerseries](#)

or

<https://cran.r-project.org/web/packages/hypergeo/hypergeo.pdf>

---

[mle2](#)

[mle2-class](#)

or

<https://cran.r-project.org/web/packages/bbmle/bbmle.pdf>

## Examples

```
No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies
#estimating the parameters using maximum log likelihood value and assigning it
parameters=bbmle::mle2(EstMLEGHGBB,start = list(a=0.1,b=0.1,c=0.2),data = list(x=No.D.D,freq=Obs.fre.1))
bbmle::coef(parameters)      #extracting the parameters
aGHGBB=bbmle::coef(parameters)[1]  #assigning the estimated a
bGHGBB=bbmle::coef(parameters)[2]  #assigning the estimated b
cGHGBB=bbmle::coef(parameters)[3]  #assigning the estimated c

#fitting when the random variable,frequencies,shape parameter values are given.
fitGHGBB(No.D.D,Obs.fre.1,aGHGBB,bGHGBB,cGHGBB)

fitGHGBB(No.D.D,Obs.fre.1,aGHGBB,bGHGBB,cGHGBB,F)$exp.freq      #extracting the expected frequencies
```

---

[fitKumBin](#)

*Fitting the Kumaraswamy Binomial Distribution when binomial random variable, frequency and shape parameters a and b, iterations parameter it are given*

---

## Description

The function will fit the Kumaraswamy binomial distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

## Usage

`fitKumBin(x,obs.freq,a,b,it,print)`

### Arguments

x	vector of binomial random variables
obs.freq	vector of frequencies
a	single value for shape parameter alpha representing a
b	single value for shape parameter beta representing b
it	number of iterations to converge as a proper probability function replacing infinity
print	logical value for print or not

### Details

$$0 < a, b$$

$$x = 0, 1, 2, \dots n$$

$$obs.freq \geq 0$$

$$it > 0$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of `fitKumBin` gives a list format consisting  
`bin.ran.var` binomial random variables  
`obs.freq` corresponding observed frequencies  
`exp.freq` corresponding expected frequencies  
`statistic` chi-squared test statistics  
`df` degree of freedom  
`p.value` probability value by chi-squared test statistic  
`over.dis.para` over dispersion value.

### References

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

### See Also

`mle2`

`mle2-class`

or

<https://cran.r-project.org/web/packages/bbmle/bbmle.pdf>

## Examples

```
No.D.D=0:7 #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies
#estimating the parameters using maximum log likelihood value and assigning it
parameters=bbmle::mle2(EstMLEKumBin,start = list(a=10.1,b=1.1,it=20000),data = list(x=No.D.D,freq=Obs.fre.1))
bbmle::coef(parameters) #extracting the parameters
aKumBin=bbmle::coef(parameters)[1] #assigning the estimated a
bKumBin=bbmle::coef(parameters)[2] #assigning the estimated b
itKumBin=bbmle::coef(parameters)[3] #assigning the estimated iterations

#fitting when the random variable,frequencies,shape parameter values are given.
fitKumBin(No.D.D,Obs.fre.1,aKumBin,bKumBin,itKumBin)

fitKumBin(No.D.D,Obs.fre.1,aKumBin,bKumBin,itKumBin,F)$exp.freq #extracting the expected frequencies
```

---

**fitMcGBB**

*Fitting the McDonald Generalized beta binomial distribution when binomial random variable, frequency and shape parameters are given*

---

## Description

The function will fit the McDonald Generalized Beta Binomial Distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

## Usage

```
fitMcGBB(x,obs.freq,a,b,c,print)
```

## Arguments

x	vector of binomial random variables
obs.freq	vector of frequencies
a	single value for shape parameter alpha representing a
b	single value for shape parameter beta representing b
c	single value for shape parameter gamma representing c
print	logical value for print or not

## Details

$$0 < a, b, c$$

$$x = 0, 1, 2, \dots$$

$$obs.freq \geq 0$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of `fitGHGBB` gives a list format consisting  
`bin.ran.var` binomial random variables  
`obs.freq` corresponding observed frequencies  
`exp.freq` corresponding expected frequencies  
`statistic` chi-squared test statistics  
`df` degree of freedom  
`p.value` probability value by chi-squared test statistic  
`over.dis.para` over dispersion value.

### References

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. International Journal of Statistics and Probability, 2(2), pp.24-41.

Available at: <http://www.ccsenet.org/journal/index.php/ijsp/article/view/23491>.

Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of McDonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.

Roozegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. Communications in Statistics - Simulation and Computation, (May), pp.0-0.

Available at: <http://www.tandfonline.com/doi/full/10.1080/03610918.2015.1088024>.

### See Also

`mle2`

`mle2-class`

or

<https://cran.r-project.org/web/packages/bbmle/bbmle.pdf>

### Examples

```
No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies
#estimating the parameters using maximum log likelihood value and assigning it
parameters=bbmle::mle2(EstMLEMcGBB,start = list(a=0.1,b=0.1,c=0.2),data = list(x=No.D.D,freq=Obs.fre.1))
aMcGBB=bbmle::coef(parameters)[1]      #assigning the estimated a
bMcGBB=bbmle::coef(parameters)[2]      #assigning the estimated b
cMcGBB=bbmle::coef(parameters)[3]      #assigning the estimated c

#fitting when the random variable,frequencies,shape parameter values are given.
fitMcGBB(No.D.D,Obs.fre.1,aMcGBB,bMcGBB,cMcGBB)

fitMcGBB(No.D.D,Obs.fre.1,aMcGBB,bMcGBB,cMcGBB,F)$exp.freq  #extracting the expected frequencies
```

**fitMultiBin**

*Fitting the Multiplicative Binomial Distribution when binomial random variable, frequency, probability of success and theta parameter are given*

**Description**

The function will fit the Multiplicative binomial distribution when random variables, corresponding frequencies, probability of success and theta parameter are given. It will provide the the expected frequencies, chi-squared test statistics value, p value and degree of freedom value so that it can be seen if this distribution fits the data.

**Usage**

```
fitMultiBin(x,obs.freq,p,theta,print)
```

**Arguments**

x	vector of binomial random variables
obs.freq	vector of frequencies
p	single value for probability of success
theta	single value for theta parameter
print	logical value for print or not

**Details**

$$obs.freq \geq 0$$

$$x = 0, 1, 2, \dots$$

$$0 < p < 1$$

$$0 < theta$$

**Value**

The output of `fitMultiBin` gives a list format consisting  
`bin.ran.var` binomial random variables  
`obs.freq` corresponding observed frequencies  
`exp.freq` corresponding expected frequencies  
`statistic` chi-squared test statistics  
`df` degree of freedom  
`p.value` probability value by chi-squared test statistic

## References

- Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.
- L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.  
Available at: <http://www.jstor.org/stable/2529589>.
- Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.  
Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990>.

## See Also

[mle2](#)  
[mle2-class](#)  
 or  
<https://cran.r-project.org/web/packages/bbmle/bbmle.pdf>

## Examples

```
No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies
#estimating the parameters using maximum log likelihood value and assigning it
parameters=bbmle::mle2(EstMLEMultiBin,start = list(p=0.1,theta=.3),data = list(x=No.D.D,freq=Obs.fre.1))
pMultiBin=bbmle::coef(parameters)[1]      #assigning the estimated probability value
thetaMultiBin=bbmle::coef(parameters)[2]  #assigning the estimated theta value

#fitting when the random variable,frequencies,probability and theta are given
fitMultiBin=fitMultiBin(No.D.D,Obs.fre.1,pMultiBin,thetaMultiBin)
fitMultiBin=fitMultiBin(No.D.D,Obs.fre.1,pMultiBin,thetaMultiBin,F)$exp.freq #extracting the expected frequencies
```

**fitTriBin**

*Fitting the Triangular Binomial Distribution when binomial random variable, frequency and mode value are given*

## Description

The function will fit the triangular binomial distribution when random variables, corresponding frequencies and mode parameter are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

## Usage

```
fitTriBin(x,obs.freq,mode,print)
```

## Arguments

x	vector of binomial random variables
obs.freq	vector of frequencies
mode	single value for mode
print	logical value for print or not

## Details

$$0 < mode = c < 1$$

$$x = 0, 1, 2, \dots$$

$$0 < mode < 1$$

$$obs.freq \geq 0$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

## Value

The output of `fitTriBin` gives a list format consisting  
`bin.ran.var` binomial random variables  
`obs.freq` corresponding observed frequencies  
`exp.freq` corresponding expected frequencies  
`statistic` chi-squared test statistics value  
`df` degree of freedom  
`p.value` probability value by chi-squared test statistic  
`over.dis.para` over dispersion value.

## References

Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.

Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In Advances in Mathematical and Statistical Modeling. Boston: Birkhäuser Boston, pp. 21-33.

Available at: [http://dx.doi.org/10.1007/978-0-8176-4626-4\\_2](http://dx.doi.org/10.1007/978-0-8176-4626-4_2).

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. British Journal of Mathematics & Computer Science, 4(24), pp.3497-3507.

Available at: <http://www.sciedomain.org/abstract.php?iid=699&id=6&aid=6427>.

## Examples

```
No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies
modeTriBin=EstMLETriBin(No.D.D,Obs.fre.1)$mode #assigning the extracted the mode value
#fitting when the random variable,frequencies,mode value are given.
fitTriBin(No.D.D,Obs.fre.1,modeTriBin)

fitTriBin(No.D.D,Obs.fre.1,modeTriBin,F)$exp.freq #extracting the expected frequencies
```

mazBETA

*Beta Distribution***Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Beta Distribution bounded between [0,1]

**Usage**

```
dBETA(p,a,b)
pBETA(p,a,b)
mazBETA(r=1,a,b)
```

**Arguments**

r	vector of moments
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
p	vector of probabilities

**Details**

The probability density function and cumulative density function of a unit bounded beta distribution with random variable P are given by

$$g_P(p) = \frac{p^{a-1}(1-p)^{b-1}}{B(a,b)}$$

$$; 0 \leq p \leq 1$$

$$G_P(p) = \frac{B_p(a,b)}{B(a,b)}$$

$$; 0 \leq p \leq 1$$

$$a, b > 0$$

The mean and the variance are denoted by

$$E[P] = \frac{a}{a+b}$$

$$var[P] = \frac{ab}{(a+b)^2(a+b+1)}$$

The moments about zero is denoted as

$$E[P^r] = \prod_{i=0}^{r-1} \left( \frac{a+i}{a+b+i} \right)$$

$$r = 1, 2, 3, \dots$$

Defined as  $B_p(a,b) = \int_0^p t^{a-1}(1-t)^{b-1} dt$  is incomplete beta integrals and  $B(a,b)$  is the beta function.

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of dBETA gives a list format consisting  
 pdf probability density values in vector form  
 mean mean of the beta distribution  
 var variance of the beta distribution  
 The output of pBETA gives the cumulative density values in vector form.  
 The output of mazBETA gives the moments about zero in vector form.

### References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley  
 Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.  
 Available at: <http://linkinghub.elsevier.com/retrieve/pii/0167947396900158>.

### See Also

[Beta](#)

or

<https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Beta.html>

### Examples

```
#plotting the random variables and probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probabiility density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),dBETA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dBETA(seq(0,1,by=0.01),2,3)$pdf    #extracting the pdf values
dBETA(seq(0,1,by=0.01),2,3)$mean   #extracting the mean
dBETA(seq(0,1,by=0.01),2,3)$var    #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),pBETA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pBETA(seq(0,1,by=0.01),2,3)    #acquiring the cumulative probability values
mazBETA(1.4,3,2)                #acquiring the moment about zero values
mazBETA(2,3,2)-mazBETA(1,3,2)^2 #acquiring the variance for a=3,b=2
mazBETA(1.9,5.5,6)              #only the integer value of moments is taken here because moments cannot be decimal
```

mazGBeta1

*Generalized Beta Type-1 Distribution***Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Generalized Beta Type-1 Distribution bounded between [0,1]

**Usage**

```
dGBeta1(p,a,b,c)
pGBeta1(p,a,b,c)
mazGBeta1(r=1,a,b,c)
```

**Arguments**

r	vector of moments
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
c	single value for shape parameter gamma representing as c
p	vector of probabilities

**Details**

The probability density function and cumulative density function of a unit bounded Generalized Beta Type-1 Distribution with random variable P are given by

$$g_P(p) = \frac{c}{B(a, b)} p^{ac-1} (1 - p^c)^{b-1}$$

$$; 0 \leq p \leq 1$$

$$G_P(p) = \frac{p^{ac}}{aB(a, b)} 2F1(a, 1 - b; p^c; a + 1)$$

$$0 \leq p \leq 1$$

$$a, b, c > 0$$

The mean and the variance are denoted by

$$E[P] = \frac{B(a + b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$var[P] = \frac{B(a + b, \frac{2}{c})}{B(a, \frac{2}{c})} - \left( \frac{B(a + b, \frac{1}{c})}{B(a, \frac{1}{c})} \right)^2$$

The moments about zero is denoted as

$$E[P^r] = \frac{B(a + b, \frac{r}{c})}{B(a, \frac{r}{c})}$$

$$r = 1, 2, 3, \dots$$

Defined as  $B(a, b)$  is beta function Defined as  $2F1(a, b; c; d)$  is Gaussian Hypergeometric function

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of `dGBeta1` gives a list format consisting  
`pdf` probability density values in vector form  
`mean` mean of the Generalized Beta Type-1 Distribution  
`var` variance of the Generalized Beta Type-1 Distribution  
The output `pGBeta1` gives the cumulative density values in vector form.  
The output `mazGBeta1` gives the moments about zero in vector form.

### References

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. International Journal of Statistics and Probability, 2(2), pp.24-41.

Available at: <http://www.ccsenet.org/journal/index.php/ijsp/article/view/23491> .

Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of McDonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.

Roozegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. Communications in Statistics - Simulation and Computation, (May), pp.0-0.

Available at: <http://www.tandfonline.com/doi/full/10.1080/03610918.2015.1088024> .

### Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
  lines(seq(0,1,by=0.001),dGBeta1(seq(0,1,by=0.001),a[i],1,2*a[i])$pdf,col = col[i])
}
dGBeta1(seq(0,1,by=0.01),2,3,1)$pdf    #extracting the pdf values
dGBeta1(seq(0,1,by=0.01),2,3,1)$mean   #extracting the mean
dGBeta1(seq(0,1,by=0.01),2,3,1)$var    #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(5)
a<-c(.001,.002,.03,1.5,2.15)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:5)
{
  lines(seq(0,1,by=0.001),pGBeta1(seq(0,1,by=0.001),a[i],1,12+a[i]),col=col[i])
}

pGBeta1(seq(0,1,by=0.01),2,3,1)  #acquiring the cumulative probability values
mazGBeta1(1.4,3,2,2)           #acquiring the moment about zero values
mazGBeta1(2,3,2,2)-mazGBeta1(1,3,2,2)^2      #acquiring the variance for a=3,b=2,c=2
mazGBeta1(3.2,3,2,2)          #only the integer value of moments is taken here because moments cannot be decimal
```

mazGHGBeta

*Gaussian Hypergeometric Generalized Beta Distribution*

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Gaussian Hypergeometric Generalized Beta distribution bounded between [0,1]

## Usage

```
dGHGBeta(p,n,a,b,c)
pGHGBeta(p,n,a,b,c)
mazGHGBeta(r=1,n,a,b,c)
```

## Arguments

r	vector of moments
n	single value for no of binomial trials
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
c	single value for shape parameter lambda representing as c
p	vector of probabilities

## Details

The probability density function and cumulative density function of a unit bounded Gaussian Hypergeometric Generalized Beta Distribution with random variable P are given by

$$g_P(p) = \frac{1}{B(a, b)} \frac{2F1(-n, a; -b - n + 1; 1)}{2F1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}}$$

$$; 0 \leq p \leq 1$$

$$G_P(p) = \int_0^p \frac{1}{B(a, b)} \frac{2F1(-n, a; -b - n + 1; 1)}{2F1(-n, a; -b - n + 1; c)} t^{a-1} (1-t)^{b-1} \frac{c^{b+n}}{(c + (1-c)t)^{a+b+n}} dt$$

$$; 0 \leq p \leq 1$$

$$a, b, c > 0$$

$$n = 1, 2, 3, \dots$$

The mean and the variance are denoted by

$$E[P] = \int_0^1 \frac{p}{B(a, b)} \frac{2F1(-n, a; -b - n + 1; 1)}{2F1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp$$

$$var[P] = \int_0^1 \frac{p^2}{B(a, b)} \frac{2F1(-n, a; -b - n + 1; 1)}{2F1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp - (E[p])^2$$

The moments about zero is denoted as

$$E[P^r] = \int_0^1 \frac{p^r}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}} dp$$

$r = 1, 2, 3, \dots$

Defined as  $B(a,b)$  as the beta function Defined as  $2F1(a,b;c;d)$  as the Gaussian Hypergeometric function

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dGHGBeta gives a list format consisting

pdf probability density values in vector form

mean mean of the Gaussian Hypergeometric Generalized Beta Distribution

var variance of the Gaussian Hypergeometric Generalized Beta Distribution

The output of pGHGBeta gives the cumulative density values in vector form.

The output of mazGHGBeta give the moments about zero in vector form.

### References

Rodríguez-Avi, J., Conde-Sánchez, A., Sáez-Castillo, A. J., & Olmo-Jiménez, M. J. (2007). A generalization of the beta-binomial distribution. Journal of the Royal Statistical Society. Series C (Applied Statistics), 56(1), 51-61.

Available at : <http://dx.doi.org/10.1111/j.1467-9876.2007.00564.x>

Pearson, J., 2009. Computation of Hypergeometric Functions. Transformation, (September), p.1–123.

### See Also

[hypergeo\\_powerseries](#)

or

<https://cran.r-project.org/web/packages/hypergeo/hypergeo.pdf>

### Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
  lines(seq(0,1,by=0.001),dGHGBeta(seq(0,1,by=0.001),7,1+a[i],0.3,1+a[i])$pdf,col = col[i])
}
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$pdf  #extracting the pdf values
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$mean #extracting the mean
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$var   #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(6)
```

```

a<-c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:6)
{
lines(seq(0.01,1,by=0.001),pGHGBeta(seq(0.01,1,by=0.001),7,1+a[i],0.3,1+a[i]),col=col[i])
}

pGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659) #acquiring the cumulative probability values
mazGHGBeta(1.4,7,1.6312,0.3913,0.6659) #acquiring the moment about zero values

#acquiring the variance for a=1.6312,b=0.3913,c=0.6659
mazGHGBeta(2,7,1.6312,0.3913,0.6659)-mazGHGBeta(1,7,1.6312,0.3913,0.6659)^2
mazGHGBeta(1.9,15,5,6,1) #only the integer value of moments is taken here because moments cannot be decimal

```

---

**mazKUM***Kumaraswamy Distribution***Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Kumaraswamy Distribution bounded between [0,1]

**Usage**

```
dKUM(p,a,b)
pKUM(p,a,b)
mazKUM(r=1,a,b)
```

**Arguments**

r	vector of moments
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
p	vector of probabilities

**Details**

The probability density function and cumulative density function of a unit bounded Kumaraswamy Distribution with random variable P are given by

$$g_P(p) = abp^{a-1}(1-p^a)^{b-1}$$

$$; 0 \leq p \leq 1$$

$$G_P(p) = 1 - (1-p^a)^b$$

$$; 0 \leq p \leq 1$$

$$a, b > 0$$

The mean and the variance are denoted by

$$E[P] = bB\left(1 + \frac{1}{a}, b\right)$$

$$var[P] = bB\left(1 + \frac{2}{a}, b\right) - \left(bB\left(1 + \frac{1}{a}, b\right)\right)^2$$

The moments about zero is denoted as

$$E[P^r] = bB\left(1 + \frac{r}{a}, b\right)$$

$r = 1, 2, 3, \dots$

Defined as  $B(a, b)$  is the beta function.

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of `dKUM` gives a list format consisting  
`pdf` probability density values in vector form  
`mean` mean of the kumaraswamy distribution  
`var` variance of the kumaraswamy distribution  
The output of `pKUM` gives the cumulative density values in vector form.  
The output of `mazKUM` gives the moments about zero in vector form.

### References

- Kumaraswamy, P. (1980). A generalized probability density function for double-bounded random processes. *Journal of Hydrology*, 46(1), 79-88.  
Available at : [http://dx.doi.org/10.1016/0022-1694\(80\)90036-0](http://dx.doi.org/10.1016/0022-1694(80)90036-0)
- Jones, M. C. (2009). Kumaraswamy's distribution: A beta-type distribution with some tractability advantages. *Statistical Methodology*, 6(1), 70-81.  
Available at : <http://dx.doi.org/10.1016/j.stamet.2008.04.001>

### See Also

[Kumaraswamy](#)

or

<https://cran.r-project.org/web/packages/extrDistr/extrDistr.pdf>

### Examples

```
#plotting the random variables and probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,6))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),dKUM(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}
```

```

dKUM(seq(0,1,by=0.01),2,3)$pdf  #extracting the probability values
dKUM(seq(0,1,by=0.01),2,3)$mean #extracting the mean
dKUM(seq(0,1,by=0.01),2,3)$var  #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),pKUM(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}
pKUM(seq(0,1,by=0.01),2,3)    #acquiring the cumulative probability values
mazKUM(1.4,3,2)              #acquiring the moment about zero values
mazKUM(2,2,3)-mazKUM(1,2,3)^2 #acquiring the variace for a=2,b=3
mazKUM(1.9,5.5,6)            #only the integer value of moments is taken here because moments cannot be decimal

```

**mazTRI***Triangular Distribution bounded between [0,1]***Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Triangular Distribution bounded between [0,1]

**Usage**

```
dTRI(p,mode)
pTRI(p,mode)
mazTRI(r=1,mode)
```

**Arguments**

<i>r</i>	vector of moments
<i>mode</i>	single value for mode
<i>p</i>	vector of probabilities

**Details**

Setting  $\min = 0$  and  $\max = 1$   $mode = c$  in the triangular distribution a unit bounded triangular distribution can be obtained. The probability density function and cumulative density function of a unit bounded triangular distribution with random variable P are given by

$$g_P(p) = \frac{2p}{c}$$

$$; 0 \leq p < c$$

$$g_P(p) = \frac{2(1-p)}{(1-c)}$$

;  $c \leq p \leq 1$

$$G_P(p) = \frac{p^2}{c}$$

;  $0 \leq p < c$

$$G_P(p) = 1 - \frac{(1-p)^2}{(1-c)}$$

;  $c \leq p \leq 1$

$$0 \leq mode = c \leq 1$$

The mean and the variance are denoted by

$$E[P] = \frac{(a+b+c)}{3} = \frac{(1+c)}{3}$$

$$var[P] = \frac{a^2 + b^2 + c^2 - ab - ac - bc}{18} = \frac{(1+c^2-c)}{18}$$

Moments about zero is denoted as

$$E[P^r] = \frac{2c^{r+2}}{c(r+2)} + \frac{2(1-c^{r+1})}{(1-c)(r+1)} + \frac{2(c^{r+2}-1)}{(1-c)(r+2)}$$

$r = 1, 2, 3, \dots$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of dTRI gives a list format consisting

pdf probability density values in vector form

mean mean of the unit bounded triangular distribution

variance variance of the unit bounded triangular distribution

The output of pTRI gives the cumulative density values in vector form.

The output of mazTRI give the moments about zero in vector form.

### References

Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley

Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In Advances in Mathematical and Statistical Modeling. Boston: Birkhäuser Boston, pp. 21-33.

Available at: [http://dx.doi.org/10.1007/978-0-8176-4626-4\\_2](http://dx.doi.org/10.1007/978-0-8176-4626-4_2).

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. British Journal of Mathematics & Computer Science, 4(24), pp.3497-3507.

Available at: <http://www.sciedomain.org/abstract.php?iid=699&id=6&aid=6427>.

**See Also**

[triangle](#)

or

<https://cran.r-project.org/web/packages/triangle/triangle.pdf>

[Triangular](#)

or

<https://cran.r-project.org/web/packages/extrDistr/extrDistr.pdf>

**Examples**

```
#plotting the random variables and probability values
col<-rainbow(4)
x<-seq(0.2,0.8,by=0.2)
plot(0,0,main="Probability density graph",xlab="Random variable",
ylab="Probability density values",xlim = c(0,1),ylim = c(0,3))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),dTRI(seq(0,1,by=0.01),x[i])$pdf,col = col[i])
}

dTRI(seq(0,1,by=0.05),0.3)$pdf      #extracting the pdf values
dTRI(seq(0,1,by=0.01),0.3)$mean    #extracting the mean
dTRI(seq(0,1,by=0.01),0.3)$var     #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(4)
x<-seq(0.2,0.8,by=0.2)
plot(0,0,main="Cumulative density graph",xlab="Random variable",
ylab="Cumulative density values",xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),pTRI(seq(0,1,by=0.01),x[i]),col = col[i])
}

pTRI(seq(0,1,by=0.05),0.3)      #acquiring the cumulative probability values
mazTRI(1.4,.3)                  #acquiring the moment about zero values
mazTRI(2,.3)-mazTRI(1,.3)^2    #variance for when is mode 0.3
mazTRI(1.9,0.5)                 #only the integer value of moments is taken here because moments cannot be decimal
```

**Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Uniform Distribution bounded between [0,1]

### Usage

```
dUNI(p)
pUNI(p)
mazUNI(r=1)
```

### Arguments

r	vector of moments
p	vector of probabilities

### Details

Setting  $a = 0$  and  $b = 1$  in the Uniform Distribution a unit bounded Uniform Distribution can be obtained. The probability density function and cumulative density function of a unit bounded Uniform Distribution with random variable P are given by

$$g_P(p) = 1$$

$$0 \leq p \leq 1$$

$$G_P(p) = p$$

$$0 \leq p \leq 1$$

The mean and the variance are denoted as

$$E[P] = \frac{1}{a+b} = 0.5$$

$$var[P] = \frac{(b-a)^2}{12} = 0.0833$$

Moments about zero is denoted as

$$E[P^r] = \frac{e^{rb} - e^{ra}}{r(b-a)} = \frac{e^r - 1}{r}$$

$$r = 1, 2, 3, \dots$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dUNI gives a list format consisting

pdf probability density values in vector form

mean mean of unit bounded uniform distribution

var variance of unit bounded uniform distribution

The output of pUNI gives the cumulative density values in vector form.

The output of mazUNI gives the moments about zero in vector form.

### References

Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley

**See Also**[Uniform](#)

or

<https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Uniform.html>**Examples**

```
#plotting the random variables and probability values
plot(seq(0,1,by=0.01),dUNI(seq(0,1,by=0.01))$pdf,type = "l",main="Probability density graph",
xlab="Random variable",ylab="Probability density values")
dUNI(seq(0,1,by=0.05))$pdf      #extract the pdf values
dUNI(seq(0,1,by=0.01))$mean    #extract the mean
dUNI(seq(0,1,by=0.01))$var     #extract the variance

#plotting the random variables and cumulative probability values
plot(seq(0,1,by=0.01),pUNI(seq(0,1,by=0.01)),type = "l",main="Cumulative density graph",
xlab="Random variable",ylab="Cumulative density values")
pUNI(seq(0,1,by=0.05))      #acquiring the cumulative probability values

mazUNI(c(1,2,3))      #acquiring the moment about zero values
mazUNI(1.9)           #only the integer value of moments is taken here because moments cannot be decimal
```

NegLLAddBin

*Negative Log Likelihood value of Additive Binomial distribution***Description**

This function will calculate the negative log likelihood value when the vector of binomial random variable and vector of corresponding frequencies are given with the input parameters.

**Usage**

```
NegLLAddBin(x,freq,p,alpha)
```

**Arguments**

x	vector of binomial random variables
freq	vector of frequencies
p	single value for probability of success
alpha	single value for alpha parameter

**Details**

$$freq \geq 0$$

$$x = 0, 1, 2, \dots$$

$$0 < p < 1$$

$$-1 < alpha < 1$$

**Value**

The output of NegLLAddBin will produce a single numeric value

**References**

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.

Available at: <http://www.jstor.org/stable/2529589>.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990>.

Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

**Examples**

```
No.D.D=0:7           #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies
NegLLAddBin(No.D.D,Obs.fre.1,.5,.03)        #acquiring the negative log likelihood value
```

NegLLBetaBin

*Negative Log Likelihood value of Beta-Binomial Distribution*

**Description**

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters a and b.

**Usage**

```
NegLLBetaBin(x,freq,a,b)
```

**Arguments**

x	vector of binomial random variables
freq	vector of frequencies
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b

**Details**

$$0 < a, b$$

$$freq \geq 0$$

$$x = 0, 1, 2, \dots$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of NegLLBetaBin will produce a single numeric value

### References

Young-Xu, Y. & Chan, K.A., 2008. Pooling overdispersed binomial data to estimate event rate. BMC medical research methodology, 8(1), p.58.

Available at: <http://www.ncbi.nlm.nih.gov/article/2538541>&tool=pmcentrez&rendertype=abstract .

Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.

Available at: <http://linkinghub.elsevier.com/retrieve/pii/0167947396900158> .

Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. Phytopathology, 83(9), p.759.

Available at: [http://www.apsnet.org/publications/phytopathology/backissues/Documents/1993Abstracts/Phyto\\_83\\_759.htm](http://www.apsnet.org/publications/phytopathology/backissues/Documents/1993Abstracts/Phyto_83_759.htm)

### Examples

```
No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies
NegLLBetaBin(No.D.D,Obs.fre.1,.3,.4)  #acquiring the negative log likelihood value
```

NegLLCorrBin

*Negative Log Likelihood value of Correlated Binomial distribution*

### Description

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the input parameters.

### Usage

```
NegLLCorrBin(x,freq,p,cov)
```

### Arguments

x	vector of binomial random variables
freq	vector of frequencies
p	single value for probability of success
cov	single value for covariance

### Details

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$-\infty < cov < +\infty$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of NegLLCorrBin will produce a single numeric value

### References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.

Available at: <http://www.jstor.org/stable/2529589> .

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990> .

Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

### Examples

```
No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies
NegLLCorrBin(No.D.D,Obs.fre.1,.5,.03)    #acquiring the negative log likelihood value
```

NegLLGHGBB

*Negative Log Likelihood value of Gaussian Hypergeometric Generalized Beta Binomial Distribution*

### Description

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters a,b and c.

### Usage

```
NegLLGHGBB(x,freq,a,b,c)
```

### Arguments

x	vector of binomial random variables
freq	vector of frequencies
a	single value for shape parameter alpha representing a
b	single value for shape parameter beta representing b
c	single value for shape parameter lambda representing c

### Details

$$0 < a, b, c$$

$$freq \geq 0$$

$$x = 0, 1, 2, \dots$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of NegLLGHGBB will produce a single numeric value

### References

Rodríguez-Avi, J., Conde-Sánchez, A., Sáez-Castillo, A. J., & Olmo-Jiménez, M. J. (2007). A generalization of the beta-binomial distribution. Journal of the Royal Statistical Society. Series C (Applied Statistics), 56(1), 51-61.

Available at : <http://dx.doi.org/10.1111/j.1467-9876.2007.00564.x>

Pearson, J., 2009. Computation of Hypergeometric Functions. Transformation, (September), p.1–123.

### See Also

[hypergeo\\_powerseries](#)

or

<https://cran.r-project.org/web/packages/hypergeo/hypergeo.pdf>

### Examples

```
No.D.D=0:7          #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies
NegLLGHGBB(No.D.D,Obs.fre.1,.2,.3,1)     #acquiring the negative log likelihodd value
```

---

NegLLKumBin	<i>Negative Log Likelihood value of Kumaraswamy Binomial Distribution</i>
-------------	---

---

### Description

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters a and b and iterations it.

### Usage

```
NegLLKumBin(x,freq,a,b,it=25000)
```

### Arguments

x	vector of binomial random variables
freq	vector of frequencies
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
it	number of iterations to converge as a proper probability function replacing infinity

### Details

$$0 < a, b$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

$$it > 0$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of NegLLKumBin will produce a single numeric value

### References

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

### Examples

```
No.D.D=0:7          #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies
NegLLKumBin(No.D.D,Obs.fre.1,1.3,4.4) #acquiring the negative log likelihood value
```

---

NegLLMcGBB	<i>Negative Log Likelihood value of McDonald Generalized Beta Binomial Distribution</i>
------------	---

---

### Description

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters a,b and c.

### Usage

```
NegLLMcGBB(x, freq, a, b, c)
```

### Arguments

x	vector of binomial random variables
freq	vector of frequencies
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
c	single value for shape parameter gamma representing as c

### Details

$$\begin{aligned} 0 < a, b, c \\ freq \geq 0 \\ x = 0, 1, 2, \dots \end{aligned}$$

### Value

The output of NegLLMcGBB will produce a single numeric value

### References

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. International Journal of Statistics and Probability, 2(2), pp.24-41.

Available at: <http://www.ccsenet.org/journal/index.php/ijsp/article/view/23491>.

Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of McDonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.

Roozegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. Communications in Statistics - Simulation and Computation, (May), pp.0-0.

Available at: <http://www.tandfonline.com/doi/full/10.1080/03610918.2015.1088024>.

### Examples

```
No.D.D=0:7          #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies
NegLLMcGBB(No.D.D,Obs.fre.1,.2,.3,1)    #acquiring the negative log likelihood value
```

NegLLMultiBin

*Negative Log Likelihood value of Multiplicative Binomial distribution***Description**

This function will calculate the negative log likelihood value when the vector of binomial random variable and vector of corresponding frequencies are given with the input parameters.

**Usage**

```
NegLLMultiBin(x,freq,p,theta)
```

**Arguments**

x	vector of binomial random variables
freq	vector of frequencies
p	single value for probability of success
theta	single value for theta parameter

**Details**

$$freq \geq 0$$

$$x = 0, 1, 2, \dots$$

$$0 < p < 1$$

$$0 < theta$$

**Value**

The output of NegLLMultiBin will produce a single numeric value

**References**

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. Biometrics, 34(1), pp.69-76.

Available at: <http://www.jstor.org/stable/2529589>.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990>.

**Examples**

```
No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies
NegLLMultiBin(No.D.D,Obs.fre.1,.5,3)    #acquiring the negative log likelihood value
```

**NegLLTriBin***Negative Log Likelihood value of Triangular Binomial Distribution***Description**

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the mode value.

**Usage**

```
NegLLTriBin(x, freq, mode)
```

**Arguments**

<code>x</code>	vector of binomial random variables
<code>freq</code>	vector of frequencies
<code>mode</code>	single value for mode

**Details**

$$0 < mode = c < 1$$

$$x = 0, 1, 2, \dots$$

$$freq \geq 0$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

**Value**

The output of NegLLTriBin will produce a single numeric value

**References**

Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.

Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In Advances in Mathematical and Statistical Modeling. Boston: Birkhäuser Boston, pp. 21-33.

Available at: [http://dx.doi.org/10.1007/978-0-8176-4626-4\\_2](http://dx.doi.org/10.1007/978-0-8176-4626-4_2).

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. British Journal of Mathematics & Computer Science, 4(24), pp.3497-3507.

Available at: <http://www.sciedomain.org/abstract.php?iid=699&id=6&aid=6427>.

**Examples**

```
No.D.D=0:7      #assigning the Random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies
NegLLTriBin(No.D.D,Obs.fre.1,.023)    #acquiring the Negative log likelihood value
```

**pAddBin***Additive Binomial Distribution***Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Additive Binomial Distribution.

**Usage**

```
dAddBin(x,n,p,alpha)
pAddBin(x,n,p,alpha)
```

**Arguments**

x	vector of binomial random variables
n	single value for no of binomial trials
p	single value for probability of success
alpha	single value for alpha parameter

**Details**

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values

$$P_{AddBin}(x) = \binom{n}{x} p^x (1-p)^{n-x} \left( \frac{\alpha}{2} \left( \frac{x(x-1)}{p} + \frac{(n-x)(n-x-1)}{1-p} - \frac{\alpha(n-1)}{2} \right) + 1 \right)$$

$$x = 0, 1, 2, 3, \dots n$$

$$n = 1, 2, 3, \dots$$

$$0 < p < 1$$

$$-1 < \alpha < 1$$

The mean and the variance are denoted as

$$E_{Addbin}[x] = np$$

$$Var_{Addbin}[x] = np(1-p)(1+(n-1)\alpha)$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

**Value**

The output of dAddBin gives a list format consisting

pdf probability function values in vector form

mean mean of Additive Binomial Distribution

var variance of Additive Binomial Distribution

The output of pAddBin gives cumulative probability values in vector form.

## References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.

L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.

Available at: <http://www.jstor.org/stable/2529589>.

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.

Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990>.

Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

## Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(0.58,0.59,0.6,0.61,0.62)
b<-c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Additive binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dAddBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dAddBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}
dAddBin(0:10,10,0.58,0.022)$pdf      #extracting the probability values
dAddBin(0:10,10,0.58,0.022)$mean    #extracting the mean
dAddBin(0:10,10,0.58,0.022)$var     #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(5)
a<-c(0.58,0.59,0.6,0.61,0.62)
b<-c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Additive binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
  lines(0:10,pAddBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
  points(0:10,pAddBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}
pAddBin(0:10,10,0.58,0.022)          #acquiring the cumulative probability values
```

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Beta Distribution bounded between [0,1]

**Usage**

```
dBETA(p,a,b)
pBETA(p,a,b)
mazBETA(r=1,a,b)
```

**Arguments**

p	vector of probabilities
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
r	vector of moments

**Details**

The probability density function and cumulative density function of a unit bounded beta distribution with random variable P are given by

$$g_P(p) = \frac{p^{a-1}(1-p)^{b-1}}{B(a,b)}$$

$$; 0 \leq p \leq 1$$

$$G_P(p) = \frac{B_p(a,b)}{B(a,b)}$$

$$; 0 \leq p \leq 1$$

$$a, b > 0$$

The mean and the variance are denoted by

$$E[P] = \frac{a}{a+b}$$

$$var[P] = \frac{ab}{(a+b)^2(a+b+1)}$$

The moments about zero is denoted as

$$E[P^r] = \prod_{i=0}^{r-1} \left( \frac{a+i}{a+b+i} \right)$$

$$r = 1, 2, 3, \dots$$

Defined as  $B_p(a,b) = \int_0^p t^{a-1}(1-t)^{b-1} dt$  is incomplete beta integrals and  $B(a,b)$  is the beta function.

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

**Value**

The output of dBETA gives a list format consisting

pdf probability density values in vector form

mean mean of the beta distribution

var variance of the beta distribution

The output of pBETA gives the cumulative density values in vector form.

The output of mazBETA gives the moments about zero in vector form.

## References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley

Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.

Available at: <http://linkinghub.elsevier.com/retrieve/pii/0167947396900158>.

## See Also

[Beta](#)

or

<https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Beta.html>

## Examples

```
#plotting the random variables and probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),dBETA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dBETA(seq(0,1,by=0.01),2,3)$pdf  #extracting the pdf values
dBETA(seq(0,1,by=0.01),2,3)$mean #extracting the mean
dBETA(seq(0,1,by=0.01),2,3)$var  #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),pBETA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pBETA(seq(0,1,by=0.01),2,3)  #acquiring the cumulative probability values
mazBETA(1.4,3,2)            #acquiring the moment about zero values
mazBETA(2,3,2)-mazBETA(1,3,2)^2 #acquiring the variance for a=3,b=2
mazBETA(1.9,5.5,6)          #only the integer value of moments is taken here because moments cannot be decimal
```

## Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Beta-Binomial Distribution.

**Usage**

```
dBetaBin(x,n,a,b)
pBetaBin(x,n,a,b)
```

**Arguments**

x	vector of binomial random variables
n	single value for no of binomial trials
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b

**Details**

Mixing beta distribution with binomial distribution will create the Beta-Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values

$$P_{BetaBin}(x) = \binom{n}{x} \frac{B(a+x, n+b-x)}{B(a, b)}$$

$$a, b > 0$$

$$x = 0, 1, 2, 3, \dots n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{BetaBin}[x] = \frac{na}{a+b}$$

$$Var_{BetaBin}[x] = \frac{(nab)}{(a+b)^2} \frac{(a+b+n)}{(a+b+1)}$$

$$overdispersion = \frac{1}{a+b+1}$$

Defined as  $B(a, b)$  is the beta function.

**Value**

The output of `dBetaBin` gives a list format consisting

`pdf` probability function values in vector form

`mean` mean of the Beta-Binomial Distribution

`var` variance of the Beta-Binomial Distribution

`over.dis.para` over dispersion value of the Beta-Binomial Distribution

The output of `pBetaBin` gives cumulative probability values in vector form.

## References

- Young-Xu, Y. & Chan, K.A., 2008. Pooling overdispersed binomial data to estimate event rate. BMC medical research methodology, 8(1), p.58.  
 Available at: <http://www.ncbi.nlm.nih.gov/pmc/articles/PMC2538541/>&tool=pmcentrez&rendertype=abstract .
- Trenkler, G., 1996. Continuous univariate distributions. Computational Statistics & Data Analysis, 21(1), p.119.  
 Available at: <http://linkinghub.elsevier.com/retrieve/pii/0167947396900158> .
- Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. Phytopathology, 83(9), p.759.  
 Available at: [http://www.apsnet.org/publications/phytopathology/backissues/Documents/1993Abstracts/Phyto\\_83\\_759.htm](http://www.apsnet.org/publications/phytopathology/backissues/Documents/1993Abstracts/Phyto_83_759.htm)

## Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(1,2,5,10,0.2)
plot(0,0,main="Beta-binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dBetaBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dBetaBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}

dBetaBin(0:10,10,4,.2)$pdf    #extracting the pdf values
dBetaBin(0:10,10,4,.2)$mean   #extracting the mean
dBetaBin(0:10,10,4,.2)$var     #extracting the variance
dBetaBin(0:10,10,4,.2)$over.dis para  #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
  lines(0:10,pBetaBin(0:10,10,a[i],a[i]),col = col[i])
  points(0:10,pBetaBin(0:10,10,a[i],a[i]),col = col[i])
}
pBetaBin(0:10,10,4,.2)    #acquiring the cumulative probability values
```

## Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Correlated Binomial Distribution.

**Usage**

```
dCorrBin(x,n,p,cov)
pCorrBin(x,n,p,cov)
```

**Arguments**

x	vector of binomial random variables
n	single value for no of binomial trials
p	single value for probability of success
cov	single value for covariance

**Details**

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values

$$P_{Corrbin}(x) = \binom{n}{x} (p^x)(1-p)^{n-x} \left(1 + \left(\frac{cov}{2p^2(1-p)^2}\right)((x-np)^2 + x(2p-1) - np^2)\right)$$

$$x = 0, 1, 2, 3, \dots n \quad n = 1, 2, 3, \dots \quad 0 < p < 1 \quad -\infty < cov < +\infty$$

The Correlation is inbetween

$$\frac{-2}{n(n-1)} \min\left(\frac{p}{1-p}, \frac{p}{1-p}\right) \leq cov \leq \frac{2p(1-p)}{(n-1)p(1-p) + 0.25 - fo}$$

$$\text{where } fo = \min(x - (n-1)p - 0.5)^2$$

The mean and the variance are denoted as

$$E_{Corrbin}[x] = np$$

$$Var_{Corrbin}[x] = np(1-p)(1 + (n-1)cov)$$

$$Corr_{Corrbin}[x] = \frac{cov}{p(1-p)}$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

**Value**

The output of dCorrBin gives a list format consisting

pdf probability function values in vector form

mean mean of Correlated Binomial Distribution

var variance of Correlated Binomial Distribution

corr correlation of Correlated Binomial Distribution

mincorr minimum correlation value possible

maxcorr maximum correlation value possible

The output of pCorrBin gives cumulative probability values in vector form.

## References

- Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.
- L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.  
Available at: <http://www.jstor.org/stable/2529589>.
- Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.  
Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990>.
- Jorge G. Morel and Nagaraj K. Neerchal. Overdispersion Models in SAS. SAS Institute, 2012.

## See Also

**CBprob**

or

<https://cran.r-project.org/web/packages/BinaryEPPM/BinaryEPPM.pdf>

## Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(0.58,0.59,0.6,0.61,0.62)
b<-c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dCorrBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dCorrBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}
dCorrBin(0:10,10,0.58,0.022)$pdf      #extracting the pdf values
dCorrBin(0:10,10,0.58,0.022)$mean    #extracting the mean
dCorrBin(0:10,10,0.58,0.022)$var     #extracting the variance
dCorrBin(0:10,10,0.58,0.022)$corr    #extracting the correlation
dCorrBin(0:10,10,0.58,0.022)$mincorr #extracting the minimum correlation value
dCorrBin(0:10,10,0.58,0.022)$maxcorr #extracting the maximum correlation value

#plotting the random variables and cumulative probability values
col<-rainbow(5)
a<-c(0.58,0.59,0.6,0.61,0.62)
b<-c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
  lines(0:10,pCorrBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
  points(0:10,pCorrBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}
pCorrBin(0:10,10,0.58,0.022)      #acquiring the cumulative probability values
```

pGBeta1

*Generalized Beta Type-1 Distribution***Description**

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Generalized Beta Type-1 Distribution bounded between [0,1]

**Usage**

```
dGBeta1(p,a,b,c)
pGBeta1(p,a,b,c)
mazGBeta1(r=1,a,b,c)
```

**Arguments**

p	vector of probabilities
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
c	single value for shape parameter gamma representing as c
r	vector of moments

**Details**

The probability density function and cumulative density function of a unit bounded Generalized Beta Type-1 Distribution with random variable P are given by

$$g_P(p) = \frac{c}{B(a, b)} p^{ac-1} (1 - p^c)^{b-1}$$

$$; 0 \leq p \leq 1$$

$$G_P(p) = \frac{p^{ac}}{aB(a, b)} 2F1(a, 1 - b; p^c; a + 1)$$

$$0 \leq p \leq 1$$

$$a, b, c > 0$$

The mean and the variance are denoted by

$$E[P] = \frac{B(a + b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$var[P] = \frac{B(a + b, \frac{2}{c})}{B(a, \frac{2}{c})} - \left( \frac{B(a + b, \frac{1}{c})}{B(a, \frac{1}{c})} \right)^2$$

The moments about zero is denoted as

$$E[P^r] = \frac{B(a + b, \frac{r}{c})}{B(a, \frac{r}{c})}$$

$$r = 1, 2, 3, \dots$$

Defined as  $B(a, b)$  is beta function Defined as  $2F1(a, b; c; d)$  is Gaussian Hypergeometric function

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of dGBeta1 gives a list format consisting  
pdf probability density values in vector form  
mean mean of the Generalized Beta Type-1 Distribution  
var variance of the Generalized Beta Type-1 Distribution  
The output pGBeta1 gives the cumulative density values in vector form.  
The output mazGBeta1 gives the moments about zero in vector form.

### References

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. International Journal of Statistics and Probability, 2(2), pp.24-41.

Available at: <http://www.ccsenet.org/journal/index.php/ijsp/article/view/23491> .

Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of McDonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.

Roozegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. Communications in Statistics - Simulation and Computation, (May), pp.0-0.

Available at: <http://www.tandfonline.com/doi/full/10.1080/03610918.2015.1088024> .

### Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
  lines(seq(0,1,by=0.001),dGBeta1(seq(0,1,by=0.001),a[i],1,2*a[i])$pdf,col = col[i])
}
dGBeta1(seq(0,1,by=0.01),2,3,1)$pdf    #extracting the pdf values
dGBeta1(seq(0,1,by=0.01),2,3,1)$mean   #extracting the mean
dGBeta1(seq(0,1,by=0.01),2,3,1)$var    #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(5)
a<-c(.001,.002,.03,1.5,2.15)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:5)
{
  lines(seq(0,1,by=0.001),pGBeta1(seq(0,1,by=0.001),a[i],1,12+a[i]),col=col[i])
}

pGBeta1(seq(0,1,by=0.01),2,3,1)  #acquiring the cumulative probability values
mazGBeta1(1.4,3,2,2)           #acquiring the moment about zero values
mazGBeta1(2,3,2,2)-mazGBeta1(1,3,2,2)^2      #acquiring the variance for a=3,b=2,c=2
mazGBeta1(3.2,3,2,2)          #only the integer value of moments is taken here because moments cannot be decimal
```

pGHGBB

*Gaussian Hypergeometric Generalized Beta Binomial Distribution*

## Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Gaussian Hypergeometric Generalized Beta Binomial distribution.

## Usage

```
dGHGBB(x,n,a,b,c)
pGHGBB(x,n,a,b,c)
```

## Arguments

x	vector of binomial random variables
n	single value for no of binomial trials
a	single value for shape parameter alpha value representing a
b	single value for shape parameter beta value representing b
c	single value for shape parameter lambda value representing c

## Details

Mixing Gaussian Hypergeometric Generalized Beta distribution with binomial distribution will create the Gaussian Hypergeometric Generalized Beta Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values

$$P_{GHGBB}(x) = \frac{1}{2F1(-n, a; -b - n + 1; c)} \binom{n}{x} \frac{B(x + a, n - x + b)}{B(a, b + n)} (c^x)$$

$$a, b, c > 0$$

$$x = 0, 1, 2, \dots n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{GHGBB}[x] = nE_{GHGBeta}$$

$$Var_{GHGBB}[x] = nE_{GHGBeta}(1 - E_{GHGBeta}) + n(n - 1)Var_{GHGBeta}$$

$$overdispersion = \frac{var_{GHGBeta}}{E_{GHGBeta}(1 - E_{GHGBeta})}$$

Defined as  $B(a, b)$  is the beta function. Defined as  $2F1(a, b; c; d)$  is the gaussian hypergeometric function

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dGHGBB gives a list format consisting  
 pdf probability function values in vector form  
 mean mean of Gaussian Hypergeometric Generalized Beta Binomial Distribution  
 var variance of Gaussian Hypergeometric Generalized Beta Binomial Distribution  
 over.dis.para over dispersion value of Gaussian Hypergeometric Generalized Beta Binomial Dis-  
 tribution

The output of pGHGBB gives cumulative probability function values in vector form

### References

- Rodríguez-Avi, J., Conde-Sánchez, A., Sáez-Castillo, A. J., & Olmo-Jiménez, M. J. (2007). A generalization of the beta-binomial distribution. Journal of the Royal Statistical Society. Series C (Applied Statistics), 56(1), 51-61.
- Available at : <http://dx.doi.org/10.1111/j.1467-9876.2007.00564.x>
- Pearson, J., 2009. Computation of Hypergeometric Functions. Transformation, (September), p.1–123.

### See Also

[hypergeo\\_powerseries](#)  
 or  
<https://cran.r-project.org/web/packages/hypergeo/hypergeo.pdf>

### Examples

```
#plotting the random variables and probability values
col<-rainbow(6)
a<-c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="GHGBB probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,7),ylim = c(0,0.9))
for (i in 1:6)
{
  lines(0:7,dGHGBB(0:7,7,1+a[i],0.3,1+a[i])$pdf,col = col[i],lwd=2.85)
  points(0:7,dGHGBB(0:7,7,1+a[i],0.3,1+a[i])$pdf,col = col[i],pch=16)
}
dGHGBB(0:7,7,1.3,0.3,1.3)$pdf      #extracting the pdf values
dGHGBB(0:7,7,1.3,0.3,1.3)$mean    #extracting the mean
dGHGBB(0:7,7,1.3,0.3,1.3)$var     #extracting the variance
dGHGBB(0:7,7,1.3,0.3,1.3)$over.dis.par #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,7),ylim = c(0,1))
for (i in 1:4)
{
  lines(0:7,pGHGBB(0:7,7,1+a[i],0.3,1+a[i]),col = col[i])
  points(0:7,pGHGBB(0:7,7,1+a[i],0.3,1+a[i]),col = col[i])
}
```

```
pGHGBB(0:7,7,1.3,0.3,1.3)      #acquiring the cumulative probability values
```

**pGHGBeta**

*Gaussian Hypergeometric Generalized Beta Distribution*

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Gaussian Hypergeometric Generalized Beta distribution bounded between [0,1]

## Usage

```
dGHGBeta(p,n,a,b,c)
pGHGBeta(p,n,a,b,c)
mazGHGBeta(r=1,n,a,b,c)
```

## Arguments

p	vector of probabilities
n	single value for no of binomial trials
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
c	single value for shape parameter lambda representing as c
r	vector of moments

## Details

The probability density function and cumulative density function of a unit bounded Gaussian Hypergeometric Generalized Beta Distribution with random variable P are given by

$$g_P(p) = \frac{1}{B(a, b)} \frac{2F1(-n, a; -b - n + 1; 1)}{2F1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}}$$

$$; 0 \leq p \leq 1$$

$$G_P(p) = \int_0^p \frac{1}{B(a, b)} \frac{2F1(-n, a; -b - n + 1; 1)}{2F1(-n, a; -b - n + 1; c)} t^{a-1} (1-t)^{b-1} \frac{c^{b+n}}{(c + (1-c)t)^{a+b+n}} dt$$

$$; 0 \leq p \leq 1$$

$$a, b, c > 0$$

$$n = 1, 2, 3, \dots$$

The mean and the variance are denoted by

$$E[P] = \int_0^1 \frac{p}{B(a, b)} \frac{2F1(-n, a; -b - n + 1; 1)}{2F1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp$$

$$var[P] = \int_0^1 \frac{p^2}{B(a, b)} \frac{2F1(-n, a; -b - n + 1; 1)}{2F1(-n, a; -b - n + 1; c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c + (1-c)p)^{a+b+n}} dp - (E[p])^2$$

The moments about zero is denoted as

$$E[P^r] = \int_0^1 \frac{p^r}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1} (1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}} dp$$

$r = 1, 2, 3, \dots$

Defined as  $B(a,b)$  as the beta function Defined as  $2F1(a,b;c;d)$  as the Gaussian Hypergeometric function

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dGHGBeta gives a list format consisting

pdf probability density values in vector form

mean mean of the Gaussian Hypergeometric Generalized Beta Distribution

var variance of the Gaussian Hypergeometric Generalized Beta Distribution

The output of pGHGBeta gives the cumulative density values in vector form.

The output of mazGHGBeta give the moments about zero in vector form.

### References

Rodríguez-Avi, J., Conde-Sánchez, A., Sáez-Castillo, A. J., & Olmo-Jiménez, M. J. (2007). A generalization of the beta-binomial distribution. Journal of the Royal Statistical Society. Series C (Applied Statistics), 56(1), 51-61.

Available at : <http://dx.doi.org/10.1111/j.1467-9876.2007.00564.x>

Pearson, J., 2009. Computation of Hypergeometric Functions. Transformation, (September), p.1–123.

### See Also

[hypergeo\\_powerseries](#)

or

<https://cran.r-project.org/web/packages/hypergeo/hypergeo.pdf>

### Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
  lines(seq(0,1,by=0.001),dGHGBeta(seq(0,1,by=0.001),7,1+a[i],0.3,1+a[i])$pdf,col = col[i])
}
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$pdf #extracting the pdf values
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$mean #extracting the mean
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$var #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(6)
```

```

a<-c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:6)
{
  lines(seq(0.01,1,by=0.001),pGHGBeta(seq(0.01,1,by=0.001),7,1+a[i],0.3,1+a[i]),col=col[i])
}

pGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659) #acquiring the cumulative probability values
mazGHGBeta(1.4,7,1.6312,0.3913,0.6659)           #acquiring the moment about zero values

#acquiring the variance for a=1.6312,b=0.3913,c=0.6659
mazGHGBeta(2,7,1.6312,0.3913,0.6659)-mazGHGBeta(1,7,1.6312,0.3913,0.6659)^2
mazGHGBeta(1.9,15,5,6,1) #only the integer value of moments is taken here because moments cannot be decimal

```

---

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Kumaraswamy Distribution bounded between [0,1]

## Usage

```

dKUM(p,a,b)
pKUM(p,a,b)
mazKUM(r=1,a,b)

```

## Arguments

p	vector of probabilities
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
r	vector of moments

## Details

The probability density function and cumulative density function of a unit bounded Kumaraswamy Distribution with random variable P are given by

$$g_P(p) = abp^{a-1}(1-p^a)^{b-1}$$

$$; 0 \leq p \leq 1$$

$$G_P(p) = 1 - (1-p^a)^b$$

$$; 0 \leq p \leq 1$$

$$a, b > 0$$

The mean and the variance are denoted by

$$E[P] = bB\left(1 + \frac{1}{a}, b\right)$$

$$var[P] = bB\left(1 + \frac{2}{a}, b\right) - \left(bB\left(1 + \frac{1}{a}, b\right)\right)^2$$

The moments about zero is denoted as

$$E[P^r] = bB\left(1 + \frac{r}{a}, b\right)$$

$r = 1, 2, 3, \dots$

Defined as  $B(a, b)$  is the beta function.

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of `dKUM` gives a list format consisting  
`pdf` probability density values in vector form  
`mean` mean of the kumaraswamy distribution  
`var` variance of the kumaraswamy distribution  
The output of `pKUM` gives the cumulative density values in vector form.  
The output of `mazKUM` gives the moments about zero in vector form.

### References

- Kumaraswamy, P. (1980). A generalized probability density function for double-bounded random processes. *Journal of Hydrology*, 46(1), 79-88.  
Available at : [http://dx.doi.org/10.1016/0022-1694\(80\)90036-0](http://dx.doi.org/10.1016/0022-1694(80)90036-0)
- Jones, M. C. (2009). Kumaraswamy's distribution: A beta-type distribution with some tractability advantages. *Statistical Methodology*, 6(1), 70-81.  
Available at : <http://dx.doi.org/10.1016/j.stamet.2008.04.001>

### See Also

[Kumaraswamy](#)

or

<https://cran.r-project.org/web/packages/extrDistr/extrDistr.pdf>

### Examples

```
#plotting the random variables and probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,6))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),dKUM(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}
```

```

dKUM(seq(0,1,by=0.01),2,3)$pdf   #extracting the probability values
dKUM(seq(0,1,by=0.01),2,3)$mean  #extracting the mean
dKUM(seq(0,1,by=0.01),2,3)$var   #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),pKUM(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}
pKUM(seq(0,1,by=0.01),2,3)    #acquiring the cumulative probability values
mazKUM(1.4,3,2)              #acquiring the moment about zero values
mazKUM(2,2,3)-mazKUM(1,2,3)^2 #acquiring the variace for a=2,b=3
mazKUM(1.9,5.5,6)            #only the integer value of moments is taken here because moments cannot be decimal

```

**pKumBin***Kumaraswamy Binomial Distribution***Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Kumaraswamy Binomial Distribution.

**Usage**

```
dKumBin(x,n,a,b,it=25000)
pKumBin(x,n,a,b,it=25000)
```

**Arguments**

<i>x</i>	vector of binomial random variables
<i>n</i>	single value for no of binomial trial
<i>a</i>	single value for shape parameter alpha representing a
<i>b</i>	single value for shape parameter beta representing b
<i>it</i>	number of iterations to converge as a proper probability function replacing infinity

**Details**

Mixing kumaraswamy distribution with binomial distribution will create the Kumaraswamy Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values

$$P_{KumBin}(x) = ab \binom{n}{x} \sum_{j=0}^{it} (-1)^j \binom{b-1}{j} B(x + a + aj, n - x + 1)$$

$$a, b > 0$$

$$x = 0, 1, 2, \dots n$$

$$n = 1, 2, 3, \dots$$

$$it > 0$$

The mean, variance and over dispersion are denoted as

$$E_{KumBin}[x] = nbB\left(1 + \frac{1}{a}, b\right)$$

$$Var_{KumBin}[x] = (n^2)b\left(B\left(1 + \frac{2}{a}, b\right) - bB\left(1 + \frac{1}{a}, b\right)^2\right) + nb\left(B\left(1 + \frac{1}{a}, b\right) - B\left(1 + \frac{2}{a}, b\right)\right)$$

$$overdispersion = \frac{(bB\left(1 + \frac{2}{a}, b\right) - (bB\left(1 + \frac{1}{a}, b\right))^2)}{(bB\left(1 + \frac{1}{a}, b\right) - (bB\left(1 + \frac{1}{a}, b\right))^2)}$$

Defined as  $B(a, b)$  is the beta function

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of dKumBin gives a list format consisting

pdf probability function values in vector form

mean mean of the Kumaraswamy Binomial Distribution

var variance of the Kumaraswamy Binomial Distribution

over.dis.para over dispersion value of the Kumaraswamy Distribution

The output of pKumBin gives cumulative probability values in vector form.

### References

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

### Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(1,2,5,10,.85)
plot(0,0,main="Kumaraswamy binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dKumBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dKumBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}
dKumBin(0:10,10,4,2)$pdf #extracting the pdf values
dKumBin(0:10,10,4,2)$mean #extracting the mean
dKumBin(0:10,10,4,2)$var #extracting the variance
dKumBin(0:10,10,4,2)$over.dis.para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col<-rainbow(5)
```

```

a<-c(1,2,5,10,.85)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
  lines(0:10,pKumBin(0:10,10,a[i],a[i]),col = col[i])
  points(0:10,pKumBin(0:10,10,a[i],a[i]),col = col[i])
}
pKumBin(0:10,10,4,2)    #acquiring the cumulative probability values

```

---

Plant\_Disease\_data      *Plant disease incidence data*

---

### Description

Cochran(1936) provided a data that comprise the number of tomato spotted wilt virus(TSWV) infected tomato plants in the field trials in Australia. The field map was divided into 160 'quadrats'. 9 tomato plants in each quadrat. then the numbers of TSWV infected tomato plants were counted in each quadrat. Number of infected plants out of 9 plants per quadrat can be treated as a binomial variable. the collection of all such responses from all 160 quadrats would form "binomial outcome data" below provided is a data set similar to cochran plant disease incidence data.

### Usage

Plant\_Disease\_data

### Format

A data frame with 2 variables and 10 observations

Dis.plant	Diseased Plants
fre	Observed frequencies

### Details

Marcus R(1984). orange trees infected with citrus tristeza virus (CTV) in an orchard in central Israel. We divided the field map into 84 "quadrats" of 4 rows x 3 columns and counted the total number (1981 + 1982) of infected trees out of a maximum of n = 12 in each quadrat

### Source

Extracted from

Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. *Phytopathology*, 83(9), p.759.

Available at: [http://www.apsnet.org/publications/phytopathology/backissues/Documents/1993Abstracts/Phyto\\_83\\_759.htm](http://www.apsnet.org/publications/phytopathology/backissues/Documents/1993Abstracts/Phyto_83_759.htm).

### Examples

```

Plant_Disease_data$Dis.plant      # extracting the binomial random variables
sum(Plant_Disease_data$fre)       # summing all the frequencies

```

## Description

These functions provide the ability for generating probability function values and cumulative probability function values for the McDonald Generalized Beta Binomial Distribution.

## Usage

```
dMcGBB(x, n, a, b, c)
pMcGBB(x, n, a, b, c)
```

## Arguments

x	vector of binomial random variables
n	single value for no of binomial trials
a	single value for shape parameter alpha representing as a
b	single value for shape parameter beta representing as b
c	single value for shape parameter gamma representing as c

## Details

Mixing Generalized Beta Type-1 Distribution with binomial distribution the probability function value and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values

$$P_{McGBB}(x) = \binom{n}{x} \frac{1}{B(a, b)} \left( \sum_{j=0}^{n-x} (-1)^j \binom{n-x}{j} B\left(\frac{x}{c} + a + \frac{j}{c}, b\right) \right)$$

$$a, b, c > 0$$

The mean, variance and over dispersion are denoted as

$$E_{McGBB}[x] = n \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$Var_{McGBB}[x] = n^2 \left( \frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - \left( \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} \right)^2 \right) + n \left( \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} - \frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} \right)$$

$$overdispersion = \frac{\frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - \left( \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} \right)^2}{\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} - \left( \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} \right)^2}$$

$$x = 0, 1, 2, \dots, n$$

$$n = 1, 2, 3, \dots$$

### Value

The output of `dMcGBB` gives a list format consisting  
`pdf` probability function values in vector form  
`mean` mean of McDonald Generalized Beta Binomial Distribution  
`var` variance of McDonald Generalized Beta Binomial Distribution  
`over.dis.para` over dispersion value of McDonald Generalized Beta Binomial Distribution  
The output of `pMcGBB` gives cumulative probability function values in vector form

### References

- Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. International Journal of Statistics and Probability, 2(2), pp.24-41.  
Available at: <http://www.ccsenet.org/journal/index.php/ijsp/article/view/23491>.
- Janiffer, N.M., Islam, A. & Luke, O., 2014. Estimating Equations for Estimation of McDonald Generalized Beta - Binomial Parameters. , (October), pp.702-709.
- Roozegar, R., Tahmasebi, S. & Jafari, A.A., 2015. The McDonald Gompertz Distribution: Properties and Applications. Communications in Statistics - Simulation and Computation, (May), pp.0-0.  
Available at: <http://www.tandfonline.com/doi/full/10.1080/03610918.2015.1088024>.

### Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(1,2,5,10,0.6)
plot(0,0,main="McDonald generalized beta-binomial probability function graph",
xlab="Binomial random variable",ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dMcGBB(0:10,10,a[i],2.5,a[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dMcGBB(0:10,10,a[i],2.5,a[i])$pdf,col = col[i],pch=16)
}
dMcGBB(0:10,10,4,2,1)$pdf           #extracting the pdf values
dMcGBB(0:10,10,4,2,1)$mean         #extracting the mean
dMcGBB(0:10,10,4,2,1)$var          #extracting the variance
dMcGBB(0:10,10,4,2,1)$over.dis.para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col<-rainbow(4)
a<-c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
  lines(0:10,pMcGBB(0:10,10,a[i],a[i],2),col = col[i])
  points(0:10,pMcGBB(0:10,10,a[i],a[i],2),col = col[i])
}
pMcGBB(0:10,10,4,2,1)      #acquiring the cumulative probability values
```

**pMultiBin***Multiplicative Binomial Distribution*

### Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Multiplicative Binomial Distribution.

### Usage

```
dMultiBin(x,n,p,theta)
pMultiBin(x,n,p,theta)
```

### Arguments

x	vector of binomial random variables
n	single value for no of binomial trials
p	single value for probability of success
theta	single value for theta

### Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values

$$P_{MultiBin}(x) = \binom{n}{x} p^x (1-p)^{n-x} \frac{(theta^{x(n-x)})}{f(p, theta, n)}$$

here  $f(p, theta, n)$  is

$$f(p, theta, n) = \sum_{k=0}^n \binom{n}{k} p^k (1-p)^{n-k} (theta^{k(n-k)})$$

$$x = 0, 1, 2, 3, \dots, n$$

$$n = 1, 2, 3, \dots$$

$$k = 0, 1, 2, \dots, n$$

$$0 < p < 1$$

$$0 < theta$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of dMultiBin gives a list format consisting  
pdf probability function values in vector form

The output of pMultiBin gives cumulative probability values in vector form.

## References

- Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate discrete distributions (Vol. 444). Hoboken, NJ: Wiley-Interscience.
- L. L. Kupper, J.K.H., 1978. The Use of a Correlated Binomial Model for the Analysis of Certain Toxicological Experiments. *Biometrics*, 34(1), pp.69-76.  
Available at: <http://www.jstor.org/stable/2529589>.
- Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. *Communications in Statistics - Theory and Methods*, 14(6), pp.1497-1506.  
Available at: <http://www.tandfonline.com/doi/abs/10.1080/03610928508828990>.

## Examples

```
#plotting the random variables and probability values
col<-rainbow(5)
a<-c(0.58,0.59,0.6,0.61,0.62)
b<-c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Multiplicative binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dMultiBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dMultiBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],pch=16)
}
dMultiBin(0:10,10,.58,10.022)$pdf  #extracting the pdf values

#plotting random variables and cumulative probability values
col<-rainbow(5)
a<-c(0.58,0.59,0.6,0.61,0.62)
b<-c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Multiplicative binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
  lines(0:10,pMultiBin(0:10,10,a[i],1+b[i]),col = col[i],lwd=2.85)
  points(0:10,pMultiBin(0:10,10,a[i],1+b[i]),col = col[i],pch=16)
}
pMultiBin(0:10,10,.58,10.022)      #acquiring the cumulative probability values
```

## Description

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Triangular Distribution bounded between [0,1]

## Usage

```
dTRI(p,mode)
pTRI(p,mode)
mazTRI(r=1,mode)
```

### Arguments

p	vector of probabilities
mode	single value for mode
r	vector of moments

### Details

Setting  $\min = 0$  and  $\max = 1$   $\text{mode} = c$  in the triangular distribution a unit bounded triangular distribution can be obtained. The probability density function and cumulative density function of a unit bounded triangular distribution with random variable P are given by

$$g_P(p) = \frac{2p}{c}$$

;  $0 \leq p < c$

$$g_P(p) = \frac{2(1-p)}{(1-c)}$$

;  $c \leq p \leq 1$

$$G_P(p) = \frac{p^2}{c}$$

;  $0 \leq p < c$

$$G_P(p) = 1 - \frac{(1-p)^2}{(1-c)}$$

;  $c \leq p \leq 1$

$$0 \leq \text{mode} = c \leq 1$$

The mean and the variance are denoted by

$$E[P] = \frac{(a+b+c)}{3} = \frac{(1+c)}{3}$$

$$\text{var}[P] = \frac{a^2 + b^2 + c^2 - ab - ac - bc}{18} = \frac{(1+c^2-c)}{18}$$

Moments about zero is denoted as

$$E[P^r] = \frac{2c^{r+2}}{c(r+2)} + \frac{2(1-c^{r+1})}{(1-c)(r+1)} + \frac{2(c^{r+2}-1)}{(1-c)(r+2)}$$

$r = 1, 2, 3, \dots$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of dTRI gives a list format consisting

pdf probability density values in vector form

mean mean of the unit bounded triangular distribution

variance variance of the unit bounded triangular distribution

The output of pTRI gives the cumulative density values in vector form.

The output of mazTRI give the moments about zero in vector form.

## References

- Horsnell, G. (1957). Economic acceptance sampling schemes. *Journal of the Royal Statistical Society, Series A*, 120:148-191.
- Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley
- Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In Advances in Mathematical and Statistical Modeling. Boston: Birkhäuser Boston, pp. 21-33.
- Available at: [http://dx.doi.org/10.1007/978-0-8176-4626-4\\_2](http://dx.doi.org/10.1007/978-0-8176-4626-4_2).
- Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. *British Journal of Mathematics & Computer Science*, 4(24), pp.3497-3507.
- Available at: <http://www.sciencedomain.org/abstract.php?iid=699&id=6&aid=6427>.

## See Also

[triangle](#)

or

<https://cran.r-project.org/web/packages/triangle/triangle.pdf>

---

[Triangular](#)

or

<https://cran.r-project.org/web/packages/extrDistr/extrDistr.pdf>

## Examples

```
#plotting the random variables and probability values
col<-rainbow(4)
x<-seq(0.2,0.8,by=0.2)
plot(0,0,main="Probability density graph",xlab="Random variable",
ylab="Probability density values",xlim = c(0,1),ylim = c(0,3))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),dTRI(seq(0,1,by=0.01),x[i])$pdf,col = col[i])
}

dTRI(seq(0,1,by=0.05),0.3)$pdf      #extracting the pdf values
dTRI(seq(0,1,by=0.01),0.3)$mean    #extracting the mean
dTRI(seq(0,1,by=0.01),0.3)$var     #extracting the variance

#plotting the random variables and cumulative probability values
col<-rainbow(4)
x<-seq(0.2,0.8,by=0.2)
plot(0,0,main="Cumulative density graph",xlab="Random variable",
ylab="Cumulative density values",xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
  lines(seq(0,1,by=0.01),pTRI(seq(0,1,by=0.01),x[i]),col = col[i])
}

pTRI(seq(0,1,by=0.05),0.3)          #acquiring the cumulative probability values
mazTRI(1.4,.3)                     #acquiring the moment about zero values
```

---

```
mazTRI(2,.3)-mazTRI(1,.3)^2      #variance for when is mode 0.3
mazTRI(1.9,0.5)      #only the integer value of moments is taken here because moments cannot be decimal
```

---

**pTriBin***Triangular Binomial Distribution***Description**

These functions provide the ability for generating probability function values and cumulative probability function values for the Triangular Binomial distribution.

**Usage**

```
dTriBin(x,n,mode)
pTriBin(x,n,mode)
```

**Arguments**

x	vector of binomial random variables
n	single value for no of binomial trials
mode	single value for mode

**Details**

Mixing unit bounded triangular distribution with binomial distribution will create Triangular Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values

$$P_{TriBin}(x) = 2 \binom{n}{x} (c^{-1} B_c(x+2, n-x+1) + (1-c)^{-1} B(x+1, n-x+2) - (1-c)^{-1} B_c(x+1, n-x+2))$$

$$0 < mode = c < 1$$

$$x = 0, 1, 2, \dots n$$

$$n = 1, 2, 3, \dots$$

The mean, variance and over dispersion are denoted as

$$E_{TriBin}[x] = \frac{n(1+c)}{3}$$

$$Var_{TriBin}[x] = \frac{n(n+3)}{18} - \frac{n(n-3)c(1-c)}{18}$$

$$overdispersion = \frac{(1-c+c^2)}{2(2+c-c^2)}$$

Defined as  $B_c(a, b) = \int_0^c t^{a-1} (1-t)^{b-1} dt$  is incomplete beta integrals and  $B(a, b)$  is the beta function.

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further

### Value

The output of `dTriBin` gives a list format consisting  
`pdf` probability function values in vector form  
`mean` mean of the Triangular Binomial Distribution  
`var` variance of the Triangular Binomial Distribution  
`over.dis.para` over dispersion value of the Triangular Binomial Distribution  
The output of `pTriBin` gives cumulative probability function values in vector form.

### References

- Horsnell, G. (1957). Economic acceptance sampling schemes. *Journal of the Royal Statistical Society, Series A*, 120:148-191.
- Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In Advances in Mathematical and Statistical Modeling. Boston: Birkhäuser Boston, pp. 21-33.  
Available at: [http://dx.doi.org/10.1007/978-0-8176-4626-4\\_2](http://dx.doi.org/10.1007/978-0-8176-4626-4_2).
- Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. *British Journal of Mathematics & Computer Science*, 4(24), pp.3497-3507.  
Available at: <http://www.sciedomain.org/abstract.php?iid=699&id=6&aid=6427>.

### Examples

```
#plotting the random variables and probability values
col<-rainbow(7)
x<-seq(0.1,0.7,by=0.1)
plot(0,0,main="Triangular binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,.3))
for (i in 1:7)
{
  lines(0:10,dTriBin(0:10,10,x[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dTriBin(0:10,10,x[i])$pdf,col = col[i],pch=16)
}

dTriBin(0:10,10,.4)$pdf      #extracting the pdf values
dTriBin(0:10,10,.4)$mean    #extracting the mean
dTriBin(0:10,10,.4)$var     #extracting the variance
dTriBin(0:10,10,.4)$over.dis.para #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col<-rainbow(7)
x<-seq(0.1,0.7,by=0.1)
plot(0,0,main="Triangular binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:7)
{
  lines(0:10,pTriBin(0:10,10,x[i]),col = col[i],lwd=2.85)
  points(0:10,pTriBin(0:10,10,x[i]),col = col[i],pch=16)
}
pTriBin(0:10,10,.4)    #acquiring the cumulative probability values
```

---

pUNI	<i>Uniform Distribution bounded between [0,1]</i>
------	---

---

### Description

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Uniform Distribution bounded between [0,1]

### Usage

```
dUNI(p)
pUNI(p)
mazUNI(r=1)
```

### Arguments

p	vector of probabilities
r	vector of moments

### Details

Setting  $a = 0$  and  $b = 1$  in the Uniform Distribution a unit bounded Uniform Distribution can be obtained. The probability density function and cumulative density function of a unit bounded Uniform Distribution with random variable P are given by

$$g_P(p) = 1$$

$$0 \leq p \leq 1$$

$$G_P(p) = p$$

$$0 \leq p \leq 1$$

The mean and the variance are denoted as

$$E[P] = \frac{1}{a+b} = 0.5$$

$$var[P] = \frac{(b-a)^2}{12} = 0.0833$$

Moments about zero is denoted as

$$E[P^r] = \frac{e^{rb} - e^{ra}}{r(b-a)} = \frac{e^r - 1}{r}$$

$$r = 1, 2, 3, \dots$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dUNI gives a list format consisting

pdf probability density values in vector form

mean mean of unit bounded uniform distribution

var variance of unit bounded uniform distribution

The output of pUNI gives the cumulative density values in vector form.

The output of mazUNI gives the moments about zero in vector form.

## References

- Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.
- Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, Vol. 2, Wiley Series in Probability and Mathematical Statistics, Wiley

## See Also

[Uniform](#)

or

<https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Uniform.html>

## Examples

```
#plotting the random variables and probability values
plot(seq(0,1,by=0.01),dUNI(seq(0,1,by=0.01))$pdf,type = "l",main="Probability density graph",
xlab="Random variable",ylab="Probability density values")
dUNI(seq(0,1,by=0.05))$pdf      #extract the pdf values
dUNI(seq(0,1,by=0.01))$mean    #extract the mean
dUNI(seq(0,1,by=0.01))$var     #extract the variance

#plotting the random variables and cumulative probability values
plot(seq(0,1,by=0.01),pUNI(seq(0,1,by=0.01)),type = "l",main="Cumulative density graph",
xlab="Random variable",ylab="Cumulative density values")
pUNI(seq(0,1,by=0.05))      #acquiring the cumulative probability values

mazUNI(c(1,2,3))    #acquiring the moment about zero values
mazUNI(1.9)          #only the integer value of moments is taken here because moments cannot be decimal
```

## Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Uniform Binomial Distribution.

## Usage

```
dUniBin(x,n)
pUniBin(x,n)
```

## Arguments

- |   |  |
|---|--|
| x | vector of binomial random variables    |
| n | single value for no of binomial trials |

### Details

Mixing unit bounded uniform distribution with binomial distribution will create the Uniform Binomial Distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values

$$P_{UniBin}(x) = \frac{1}{n+1}$$

$$n = 1, 2, \dots$$

$$x = 0, 1, 2, \dots n$$

The mean, variance and over dispersion are denoted as

$$E_{UniBin}[X] = \frac{n}{2}$$

$$Var_{UniBin}[X] = \frac{n(n+2)}{12}$$

$$overdispersion = \frac{1}{3}$$

**NOTE :** If input parameters are not in given domain conditions necessary error messages will be provided to go further.

### Value

The output of dUniBin gives a list format consisting

pdf probability function values in vector form

mean mean of the Uniform Binomial Distribution

var variance of the Uniform Binomial Distribution

ove.dis.para over dispersion value of Uniform Binomial Distribution

The output of pUniBin gives cumulative probability function values in vector form.

### References

Horsnell, G. (1957). Economic acceptance sampling schemes. Journal of the Royal Statistical Society, Series A, 120:148-191.

Okagbue, H. et al., 2014. Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution. British Journal of Mathematics & Computer Science, 4(24), pp.3497-3507.

Available at: <http://www.sciedomain.org/abstract.php?iid=699&id=6&aid=6427> .

### Examples

```
#plotting the binomial random variables and probability values
plot(0:10,dUniBin(0:10,10)$pdf,type="l",main="Uniform binomial probability function graph",
xlab=" Binomial random variable",ylab="Probability function values")
points(0:10,dUniBin(0:10,10)$pdf)
dUniBin(0:300,300)$pdf #extracting the pdf values
dUniBin(0:10,10)$mean #extracting the mean
dUniBin(0:10,10)$var #extracting the variance
```

```
dUniBin(0:10,10)$over.dis.para #extracting the over dispersion

#plotting the binomial random variables and cumulative probability values
plot(0:10,pUniBin(0:10,10),type="l",main="Cumulative probability function graph",
xlab=" Binomial random variable",ylab="Cumulative probability function values")
points(0:10,pUniBin(0:10,10))

pUniBin(0:15,15)      #acquiring the cumulative probability values
```

Terror\_data\_ARG

*Terror Data ARG*

## Description

Jenkins and Johnson (1975) compiled a chronology of incidents of international terrorism from 1/1968 through 04/1974. During this period 507 incidents are recorded in the world, where 64 incidents occurred in the united states and 65 ones in argentina.

## Usage

*Terror\_data\_ARG*

## Format

A data frame with 2 variables and 9 observations

Incidents No of Incidents Occured

fre Observed frequencies

## Source

Extracted from

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

## Examples

```
Terror_data_ARG$Incidents      #extracting the binomial random variables
sum(Terror_data_ARG$fre)        #summing all the frequencies
```

---

Terror\_data\_USA

*Terror Data USA*

---

### Description

Jenkins and Johnson (1975) compiled a chronology of incidents of international terrorism from 1/1968 through 04/1974. During this period 507 incidents are recorded in the world, where 64 incidents occurred in the United States and 65 ones in Argentina.

### Usage

Terror\_data\_USA

### Format

A data frame with 2 variables and 9 observations

Incidents No of Incidents Occurred

fre Observed frequencies

### Source

Extracted from

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

### Examples

```
Terror_data_USA$Incidents           #extracting the binomial random variables  
sum(Terror_data_USA$fre)            #summing all the frequencies
```

# Index

\*Topic **datasets**  
Alcohol\_data, 3  
Chromosome\_data, 4  
Course\_data, 5  
Exam\_data, 48  
Plant\_Disease\_data, 102  
Terror\_data\_ARG, 114  
Terror\_data\_USA, 115

Alcohol\_data, 3

Beta, 9, 64, 87

BODextract, 3

CBprob, 13, 91

Chromosome\_data, 4

Course\_data, 5

dAddBin, 5

dBETA, 7

dBetaBin, 9

dCorrBin, 11

dGBeta1, 14

dGHGBB, 16

dGHGBeta, 18

dKUM, 20

dKumBin, 22

dMcGBB, 24

dMultiBin, 26

dTRI, 27

dTriBin, 30

dUNI, 32

dUniBin, 33

EstMGFBetaBin, 35

EstMLEAddBin, 36

EstMLEBetaBin, 38

EstMLECorrBin, 39

EstMLEGHGBB, 41

EstMLEKumBin, 42

EstMLEMcGBB, 44

EstMLEMultiBin, 45

EstMLETriBin, 47

Exam\_data, 48

fitAddBin, 49

fitBetaBin, 50

fitBin, 52

fitCorrBin, 53

fitGHGBB, 55

fitKumBin, 56

fitMcGBB, 58

fitMultiBin, 60

fitTriBin, 61

hypergeo\_powerseries, 17, 19, 42, 56, 68, 79, 95, 97

Kumaraswamy, 21, 70, 99

mazBETA, 63

mazGBeta1, 65

mazGHGBeta, 67

mazKUM, 69

mazTRI, 71

mazUNI, 73

mle2, 36, 39, 40, 42, 43, 45, 46, 51, 56, 57, 59, 61

NegLLAddBin, 75

NegLLBetaBin, 76

NegLLCorrBin, 77

NegLLGHGBB, 78

NegLLKumBin, 80

NegLLMcGBB, 81

NegLLMultiBin, 82

NegLLTriBin, 83

pAddBin, 84

pBETA, 85

pBetaBin, 87

pCorrBin, 89

pGBeta1, 92

pGHGBB, 94

pGHGBeta, 96

pKUM, 98

pKumBin, 100

Plant\_Disease\_data, 102

pMcGBB, 103

pMultiBin, 105

pTRI, 106  
pTriBin, 109  
pUNI, 111  
pUniBin, 112  
  
Terror\_data\_ARG, 114  
Terror\_data\_USA, 115  
triangle, 29, 73, 108  
Triangular, 29, 73, 108  
  
Uniform, 33, 75, 112